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Homeworks for

Komplexitätstheorie

A. Y. 13/14

Sheet 6

Exercise 6.1 Let G be a planar graph with a vertex partition $V = V_1 \cup V_2$ such that the following holds:

- V_1 is an independent set in G.
- Every node in V_1 has at least 3 neighbors in V_2.

Show that, given these assumptions, V_2 must contain a node of degree 10 or less.

Hint 1: You may make use of the fact that every planar graph with $n \geq 4$ nodes has at least 4 nodes of degree 5 or less.

Hint 2: If there were a counterexample to our claim above, one could choose a counterexample G with $|V|$ as small as possible and, given $|V|$, $|E|$ as large as possible.

Exercise 6.2 "Planar Dominating Set (PDS)" is the special case of the problem "Dominating Set (DS)" where the input graphs have to be planar. Show that PDS is fixed-parameter tractable (with cost parameter k in the role of the fixed parameter).

Hint 1: You may make use of the results mentioned in Exercise 1.

Hint 2: The following can be shown: For any $S \subseteq V$ that does not dominate all vertices in V, there exists a node in V that is not dominated by any node in S and that has degree at most 10. You may apply this claim even if you are not able to prove it (although you should at least try to prove it).

Hint 3: Call into your mind the cool method named "Bounded Search Tree".

Exercise 6.3 Verify the hierarchy of p-grades in Fig. 11 of the Lecture Notes. You have to show that every node in the hierarchy represents a p-grade and that every edge in the hierarchy represents a $<_{pol}$-relation.

Exercise 6.4 Argue that the condition $L_1 \leq_{POL} L_2 \Rightarrow \bar{L}_1 \leq_{POL} \bar{L}_2$ is satisfied with $\leq_T$ resp. $\leq_L$ in the role of $\leq_{POL}$.