Hans U. Simon
Francesco Aldà

Bochum, November 11th 2013

Homeworks for

Komplexitätstheorie

A. Y. 13/14

Sheet 4

Correction of a mistake in the Lecture Notes: A *pseudo-polynomial reduction* from $L \subseteq \Sigma^*$ to $L' \subseteq \Sigma^*$ is a mapping $f : \Sigma^* \to \Sigma^*$ with the following properties:

a) $f(x)$ can be computed in $\text{poly}(|x|, M(x))$ steps.[1]

b) For all $x \in \Sigma^*$: $x \in L \Leftrightarrow f(x) \in L'$.

c) $|x| \leq \text{poly}(|f(x)|)$ and $M(f(x)) \leq \text{poly}(|x|, M(x))$.

Hint: In the lecture, we used the fact that the problem "Minimum Weight Perfect Matching"' can be solved in polynomial time. Similarly, you may use the fact that there exist efficient algorithms for testing whether a graph contains a perfect matching.

Exercise 4.1

a) In the lecture, we defined the problems 3-PARTITION and 4-PARTITION. Generalize these definitions (in a "natural way") so as to obtain a definition of the problem k-PARTITION for an arbitrary $k \geq 2$. (For $k = 3$ resp. $k = 4$, your general definition should collapse to the definitions that are known from the lecture.)

b) Show that, for each $k \geq 3$, k-PARTITION is strongly NP-hard by designing a pseudo-polynomial reduction from 3-PARTITION to k-PARTITION.

c) Argue why 2-PARTITION can be solved in polynomial time.

[1] $M(x)$ denotes the maximum of the numbers encoded within the input word x. If the problem L does not deal with numbers, this parameter should be ignored.

Exercise 4.2 In the lecture, you came to know the MST-based PTAA A for TSP with a performance ratio[2] of at most 2 and an extension A' of this algorithm with a performance ratio of at most 1.5.

a) Show that the performance ratio of A is not smaller than 2.

b) Show that the performance ratio of A' is not smaller than 1.5.

Exercise 4.3 BIN PACKING as an optimization problem asks for packing items of sizes $s_1, \ldots, s_n$ into as few as possible bins, taken from a possibly infinite sequence $\mathcal{B}_1, \mathcal{B}_2, \ldots$, where each bin $\mathcal{B}_j$ has a capacity of B and is empty initially. There is a PTAA named FIRST FIT (FF) for BP which works as follows: for $i = 1, \ldots, n$, put item i into the lowest-indexed bin in which it will fit (without exceeding the capacity). Show that the performance ratio of FF is at most 2.

Exercise 4.4 In the lecture, you came to know Sahni's PTA-scheme $(A_k)_{k \geq 0}$ for the problem KNAPSACK. Run A_0, A_1 and A_2 on the following instance of KNAPSACK and determine the packings that they deliver.

- $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ is the set of items.
- $B = 10$ is the knapsack capacity.
- The size function $s\colon U \to \mathbb{Z}^+$ is defined as follows: $s(u_1) = 3$, $s(u_2) = 5$, $s(u_3) = 2$, $s(u_4) = 1$, $s(u_5) = 6$, $s(u_6) = 2$.
- The value function $v\colon U \to \mathbb{Z}^+$ is defined as follows: $v(u_1) = 2$, $v(u_2) = 3$, $v(u_3) = 3$, $v(u_4) = 2$, $v(u_5) = 4$, $v(u_6) = 1$.

[2] The performance ratio of an algorithm A is the infimum over all real numbers $k \geq 1$ so that A is optimal up to a factor of k.