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Bochum, October 28th 2013

Homeworks for

Komplexitätstheorie

A. Y. 13/14

Sheet 2

Exercise 2.1

a) In the lecture we have described the codeword $\langle w_1, \ldots, w_k \rangle \in \{0,1\}^*$ of a k-tuple $(w_1, \ldots, w_k)$ such that $w_i \in \{0,1\}^*$. Let $n := \sum_{i=1}^{k} |w_i|$ denote the total length of $w_1, \ldots, w_k$. How does $|\langle w_1, \ldots, w_k \rangle|$ depend on n and k (in O-notation)?

b) Define a new codeword $C_k(w_1, \ldots, w_k) \in \{0,1\}^*$ of $(w_1, \ldots, w_k)$ inductively as follows:

$$\begin{aligned} C_1(w) &= w \\ C_{i+1}(w_1, \ldots, w_{i+1}) &= \langle C_i(w_1, \ldots, w_i), w_{i+1} \rangle \end{aligned}$$

How does $|C_k(w_1, \ldots, w_k)|$ depend on n and k (in O-notation)? How is the dependence on n if k is considered constant?

Exercise 2.2

a) Describe (in words) how a $(k+1)$-tape UTM, started on input $\alpha\$w$, simulates one step of the k-tape DTM M_α within $O(|\alpha|)$ steps. It suffices to discuss the case $k = 1$.

b) Why can the UTM achieve the same performance with k tapes provided that $k \geq 2$?

Exercise 2.3 It has been claimed in the lecture that the problem reductions in connection with TSP (showing that an efficient procedure for decision can be transformed into an efficient procedure for optimization) actually apply to any search problem $\mathcal{S}_R$ provided that R is polynomially verifiable and self-reducible and provided that the target function used in the optimization version of the problem can be evaluated in polynomial time. Sketch a proof of this claim.

Exercise 2.4

a) The problem list that was distributed in the lecture contains the problems CLIQUE and BP (among others). Define the corresponding search problems in terms of the relations R_{CLIQUE} and R_{BP}.

b) Show that both relations, R_{CLIQUE} and R_{BP}, are self-reducible.