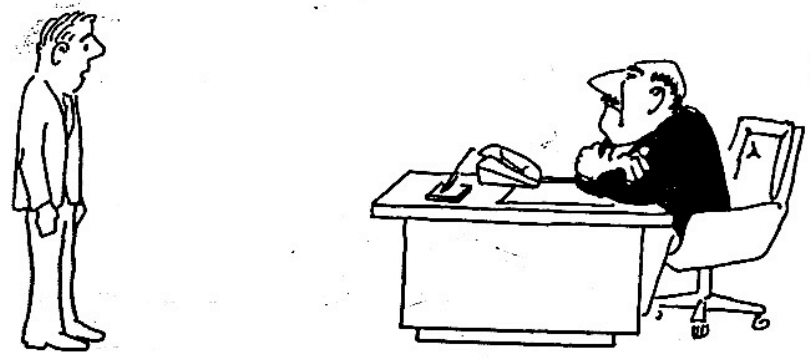


Handzettel aus: Garey & Johnson

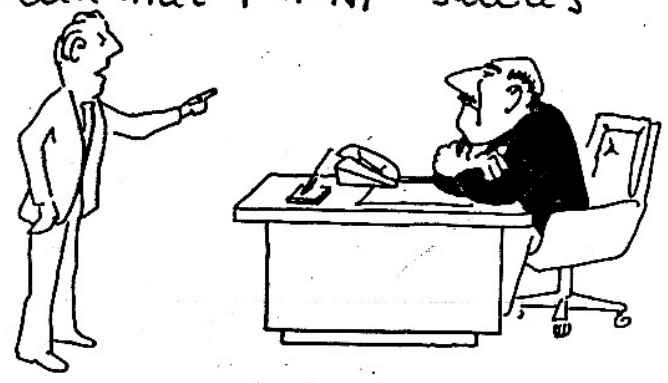
Computers and Intractability  
A Guide to the Theory of NP-Completeness  
(Freeman and Company)

I. Der ungebildete Angestellte



"I can't find an efficient algorithm, I guess I'm just too dumb."

II. Der Angestellte mit  $P \neq NP$  Beweis



"I can't find an efficient algorithm, because no such algorithm is possible!"

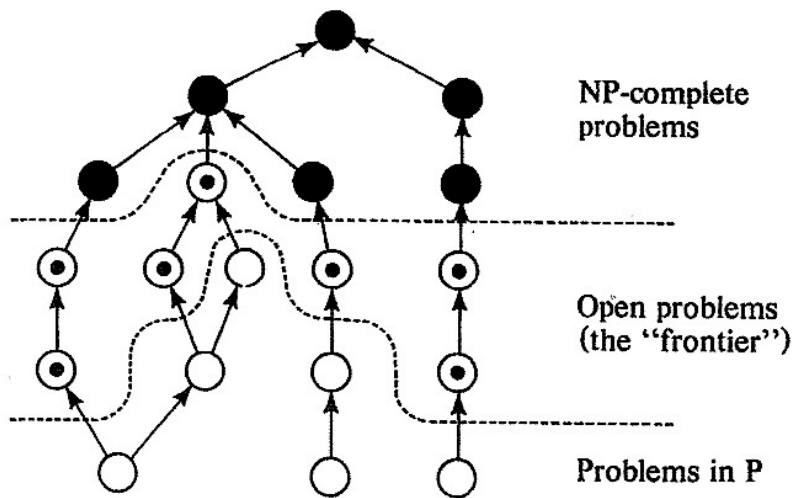
III. Der Angestellte mit NP-Vollständigkeitsbeweis



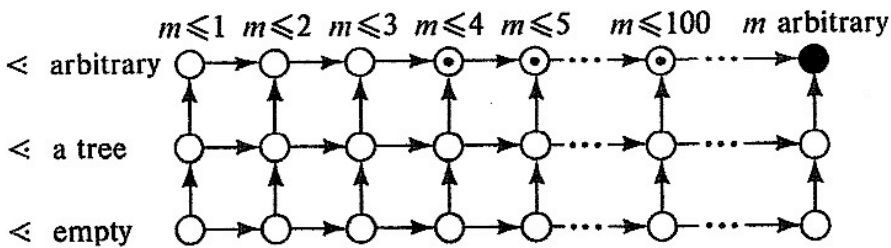
"I can't find an efficient algorithm, but neither can all these famous people."

P	NP-complete
<p><b>SHORTEST PATH BETWEEN TWO VERTICES</b>  <b>INSTANCE:</b> Graph <math>G=(V,E)</math>, length <math>l(e) \in \mathbb{Z}^+</math> for each <math>e \in E</math>, specified vertices <math>a, b \in V</math>, positive integer <math>B</math>.  <b>QUESTION:</b> Is there a simple path from <math>a</math> to <math>b</math> in <math>G</math> having total length <math>B</math> or less?</p>	<p><b>LONGEST PATH BETWEEN TWO VERTICES</b>  <b>INSTANCE:</b> Graph <math>G=(V,E)</math>, length <math>l(e) \in \mathbb{Z}^+</math> for each <math>e \in E</math>, specified vertices <math>a, b \in V</math>, positive integer <math>B</math>.  <b>QUESTION:</b> Is there a simple path from <math>a</math> to <math>b</math> in <math>G</math> having total length <math>B</math> or more?</p>
<p><b>EDGE COVER</b>  <b>INSTANCE:</b> Graph <math>G=(V,E)</math>, positive integer <math>K</math>.  <b>QUESTION:</b> Is there an <math>E' \subseteq E</math> with <math> E'  \leq K</math> such that for each <math>v \in V</math> there is some <math>e \in E'</math> for which <math>v \in e</math>?</p>	<p><b>VERTEX COVER</b>  <b>INSTANCE:</b> Graph <math>G=(V,E)</math>, positive integer <math>K</math>.  <b>QUESTION:</b> Is there a <math>V' \subseteq V</math> with <math> V'  \leq K</math> such that for each <math>e \in E</math> there is some <math>v \in V'</math> for which <math>v \in e</math>?</p>
<p><b>TRANSITIVE REDUCTION</b>  <b>INSTANCE:</b> Directed graph <math>G=(V,A)</math>, positive integer <math>K</math>.  <b>QUESTION:</b> Is there an <math>A' \subseteq V \times V</math> with <math> A'  \leq K</math> such that for all <math>u, v \in V</math> <math>G'=(V,A')</math> contains a path from <math>u</math> to <math>v</math> if and only if <math>G</math> does?</p>	<p><b>MINIMUM EQUIVALENT DIGRAPH</b>  <b>INSTANCE:</b> Directed graph <math>G=(V,A)</math>, positive integer <math>K</math>.  <b>QUESTION:</b> Is there an <math>A' \subseteq A</math> with <math> A'  \leq K</math> such that for all <math>u, v \in V</math> <math>G'=(V,A')</math> contains a path from <math>u</math> to <math>v</math> if and only if <math>G</math> does?</p>
<p><b>INTREE SCHEDULING</b>  <b>INSTANCE:</b> Set <math>T</math> of unit length tasks, deadline <math>d(t) \in \mathbb{Z}^+</math> for each <math>t \in T</math>, partial order <math>&lt;</math> on <math>T</math> such that each task has at most one immediate successor, positive integer <math>m</math>.  <b>QUESTION:</b> Can <math>T</math> be scheduled on <math>m</math> processors to obey the partial order and meet all the deadlines?</p>	<p><b>OUTTREE SCHEDULING</b>  <b>INSTANCE:</b> Set <math>T</math> of unit length tasks, deadline <math>d(t) \in \mathbb{Z}^+</math> for each <math>t \in T</math>, partial order <math>&lt;</math> on <math>T</math> such that each task has at most one immediate predecessor, positive integer <math>m</math>.  <b>QUESTION:</b> Can <math>T</math> be scheduled on <math>m</math> processors to obey the partial order and meet all the deadlines?</p>

Figure 4.1 Pairs of similar problems, one belonging to P and the other NP-complete.



**Figure 4.2** One possible state of knowledge about subproblems of an NP-complete problem  $\Pi$ . Problems are represented by circles, filled-in if known to be NP-complete, empty if known to be in P, and dotted if "open." An arrow from  $\Pi_1$  to  $\Pi_2$  signifies that  $\Pi_1$  is a subproblem of  $\Pi_2$ .



**Figure 4.3** Current state of knowledge for a collection of subproblems of PRECEDENCE CONSTRAINED SCHEDULING, using the key given in Figure 4.2.

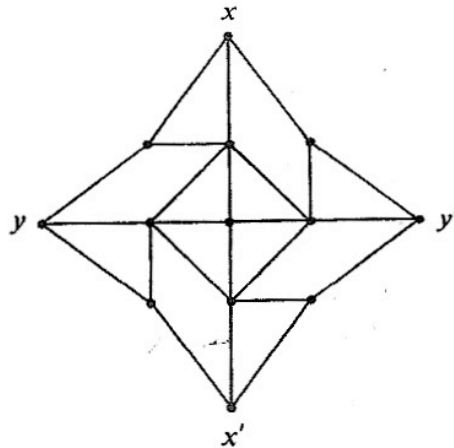


Figure 4.6 Crossover  $H$  used in the NP-completeness proof for PLANAR GRAPH 3-COLORABILITY.

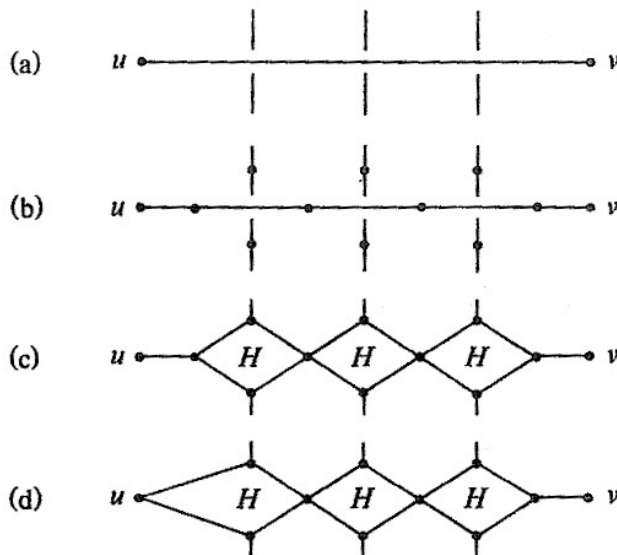


Figure 4.7 The construction of a planar graph  $G'$  from a given graph  $G$ , using the crossover  $H$  of Figure 4.6, so that 3-colorability is preserved.

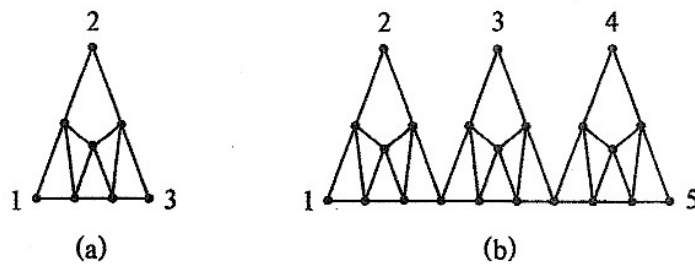


Figure 4.5 The graph  $H_3$  and vertex substitute  $H_5$  (formed from three copies of  $H_3$ ) used for proving the NP-completeness of degree-restricted GRAPH 3-COLORABILITY.