

# ENGINEERING STUDENTS AND THEIR LEARNING OF MATHEMATICS

Birgit Griese, Eva Glasmachers, Jörg Härterich, Michael Kallweit, Bettina Roesken  
Ruhr-Universität Bochum

*Students of engineering have to cope with advanced mathematics lectures even though they may not want to focus on this area of expertise. In the context of MP<sup>2</sup>, a project based at Ruhr-Universität Bochum, their learning strategies were examined. The results allow interesting insights into the level and development of crucial skills, whose transfer to the teaching of mathematics in general suggests combining mathematical topics with the teaching of learning strategies.*

## INTRODUCTION

Learning mathematics in tertiary education has received much attention in recent years, focusing both on students who are studying mathematics as major or as service subject. For instance, Guzman, Hodgson, Robert, and Villani (1998) differentiate epistemological/cognitive, sociological/cultural and didactical obstacles students are confronted with when shifting from secondary to tertiary mathematics. Regarding the former category, Guzman et al. (1998) point out that “the mathematics is different not only because the topics are different, but more to the point because of an increased depth, both with respect to the technical abilities needed to manipulate the new objects and the conceptual understanding underlying them” (p. 752). The specificities of studying mathematics at an advanced level are sometimes referred to as an “abstraction shock” that students encounter since university mathematics adds a *formal* world to the mathematics encountered at school (Tall, 2004). Some authors elaborate on motivational aspects (cf. Hoyles, Newman, & Noss, 2001) or point out the significance of affective variables (cf. Liston & O’Donoghue, 2008) while discussing the secondary-tertiary transition. However, challenges occur at many levels and require specific learning strategies students may not have developed throughout school time (Rach & Heinze, 2011).

In Germany, the high numbers of students giving up studying mathematics before graduation is alarming. In this paper, our focus is on engineering students and their learning of mathematics. At Ruhr-Universität Bochum, the project MP<sup>2</sup> was implemented, which aims at supporting students in their first year studies of engineering. In particular, the project looks into what motivates students to pursue a challenging course, which learning strategies prove successful, which cognitive dispositions and beliefs seem advantageous – and how universities can support their students. The findings presented here are part of a larger study where motivation and beliefs are investigated in detail; here the focus is on (meta)cognitive learning strategies.

## THEORETICAL BACKGROUND

We have known since the times of Piaget (Piaget, 1973) that cognitive development comes in phases even though the stages involved might not be discrete. There are different theories how these phases can be typified. “Procept” theory (Gray & Tall, 1991) stresses the duality of *process* and *concept* and its relevance for students’ understanding of mathematics. In particular, Gray and Tall (1994) “wanted to encompass the growing compressibility of knowledge characteristic of successful mathematicians. Here, not only is a single symbol viewed in a flexible way, but when the same object can be represented in different ways, these different ways are often seen as different names for the same object” (p. 120). Successful learners can change flexibly between process and concept, depending on what kind of tasks they face. Another popular theory is APOS (Dubinsky & McDonald, 2001) which distinguishes between action, process, objects and schemas and allows to describe and to evaluate learning theoretically.

Special attention has been paid to those factors that allow a teacher or lecturer to enhance learning visibly. Metacognition, knowledge about the learning process and how to modify it effectively (Flavell, 1979), plays an important role here, as well as affective variables (Liston & O’Donoghue, 2008), such as attitudes, motivation, beliefs, self-concept and approaches to learning. Where the transition to university mathematics is concerned, the difference to the mathematics taught at school and the necessity for a deeper understanding is stressed (Zucker, 1996; Engelbrecht, 2008). The learning policy followed and the individual learner types are the center of intensive research (Duffin & Simpson, 1993; Tall, 1997; Marais, 2000; Rach & Heinze, 2011; Rach & Klostermann, 2011).

The focus of this paper is on learning strategies as they represent a starting point where concise and specific measures can make a difference. Therefore the LIST questionnaire (Wild & Schiefele, 1994) for measuring learning strategies in academic studies was used. It was first compiled in the 1990s and has been modified and tested several times. It is not limited to a specific field but encompasses general items that can be applied to all kinds of subjects.

In the course of the complete study many questions about motivation, learning strategies and the efficiency of different measures are of interest. As a first step, the concentration is on three issues which represent an adequate starting point from which to evaluate the complete project:

- In what respect does a semester of university work encourage engineering students to modify their learning strategies?
- Do different interventions produce significant differences in the modulation of learning strategies? If so, how can they be described and accounted for?
- What conclusions can be drawn with respect to the teaching of mathematics, in particular to those who do not feature an inner bond with the subject?

## METHODOLOGY

### The LIST questionnaire

There are 77 questions in LIST (Wild & Schiefele, 1994) which fall into three main categories: cognitive strategies, metacognitive strategies, and resource-related strategies. As the latter are not the topic of this paper, we will concentrate on the first two. “Cognitive strategies” comprise the scales

- *organizing* ((re-)structuring the subject matter in charts, lists, groups or other arrangements, 8 items),
- *elaborating* (aiming at a deeper understanding, in particular relating new areas to those already known, 8 items),
- *critical checks* (scrutinizing statements and justifications, 8 items, see remark below), and
- *repeating* (aimed at memorizing facts, rules, and formulas, 7 items).

The “metacognitive strategies” fall into the three parts *planning*, *monitoring* and *regulating* and include 11 items altogether.

The questionnaire was adapted for our purposes in three respects: First, the original wording from 1994 did not account for the Internet, its digital resources, e-learning platforms and search engines. Today students do not bother to copy from the board during lectures, but rely on digital scripts – a technique that requires distinct skills and strategies. So whenever reference books, journals or lecture notes were mentioned in the questionnaire, their digital variants were added. A second change was necessary because of the special character of mathematics, the subject in the focus of our investigation. LIST originally contains the scale “Critical checks” which makes sense for the humanities but does not seem appropriate for first year mathematics. Therefore all items from this scale were eliminated. Third, a new starting item was added, to get in the mood and open up for the topic: “I study for my courses.” It was not used for evaluation.

A five-level Likert scale was applied to all items, consisting of *very rarely* (1), *rarely* (2), *sometimes*(3), *often* (4) and *very often* (5) in order to take into consideration which learning strategies were employed how frequently. The LIST questionnaire was given out at the beginning (pre) and at the end (post) of the first semester.

### MP<sup>2</sup> - basic assumptions and conceptual decisions

MP<sup>2</sup> is a project based at Ruhr-Universität Bochum in Germany. In order to prevent engineering students from unnecessarily failing to graduate, MP<sup>2</sup> offers support in first year mathematics. It was awarded a prize by “Stifterverband für die Deutsche Wissenschaft” for its innovative concept.

MP<sup>2</sup> is based on two assumptions (Dehling, Glasmachers, Härterich & Hellermann, 2010): First, that many students fail because of their lack of self-organization. Second, that many students lose their motivation and willingness to apply themselves because they cannot see sufficient practical application of the subject matter. MP<sup>2</sup> consists of two parts, *Math/Plus* and *Math/Practice*, which aim to tackle the two problem areas respectively. *Math/Plus* encourages and assists learning strategies in the first semester. *Math/Practice* presents an option to apply first year mathematics directly to practical engineering problems in the course of the second semester.

*Math/Plus* was set up in spring 2010 and carried through in fall and winter 2010/2011, followed by evaluation. *Math/Practice* started half a year later. Both parts will be repeated in the consequent academic year. The target group comprises first year students of engineering at Ruhr-Universität Bochum, all of whom were given the LIST questionnaire. About 1,000 of them read mathematics and obligatorily have to pass a math exam which in the past has proved a considerable obstacle.

### **Recruitment and organization**

After the first weeks of lectures and a written mini test, students had to apply for participation in *Math/Plus*. The 180 students accepted were distributed randomly among three groups. The group most attended to was the “Supported Learning Group” (SLG) which consisted of three tutorial groups of 20 students each. Another 60 students were assigned to the “Self-Directed Group” (SDG) and the “Monitored Group” (MG) respectively. As these groups experienced different core and accompanying measures, the efficiency of the single measures could be evaluated separately. In particular, without the funding it would have been difficult to implement the SLG extra tutorials. MG serves only as a control group with preconditions comparable to the other two groups’.

### **Core and accompanying measures**

Both SLG and SDG kept a daily learning diary based on Landmann and Schmitz (2007) every day. The diary contains parts “before learning” and “after learning” as well as questions about the mental state and motivation. It serves not only as a log for recording the time and learning strategies used, but also as an instrument of intervention as it aims at self-regulation (Schmitz & Wiese, 2006). The efficiency of the learning diary is the focus of another part of the evaluation of the project. Only the students in SLG met with a tutor every week for a preparatory tutorial. The meetings concentrated on a specific learning strategy, e.g. the work environment or note-taking (handwritten or digital), in combination with a mathematical problem. The students from SLG had access to a helpdesk where student assistants offered help and explanations, checked assignments and gave hints for further work. In addition to this, e-learning courses were open 24/7 for the students from SLG and SDG. They provided various test questions with immediate response, and there were filing options as well as a forum and wiki for discussion and communication.

## Procedure

Although the LIST questionnaire has been used for decades and its value and internal reliability have been tested many times, it was necessary to prove the validity of our modified questionnaire. Therefore, Cronbach's  $\alpha$  was calculated for all scales in the pre and post questionnaires in order to show that the items in each scale match up. In order to combine the different scores of items of a scale into one specific significant number, they underwent the linear transformation  $2.5 \cdot (10/n \cdot \sum x_i - 10)$  where  $n$  is the number of items in a scale, and  $x_1, \dots, x_n$  are their scores. This takes into account that there are different numbers of items in particular scales and renders numbers from 0 to 100. The Kolmogorov-Smirnov test was used to show that these transformed numbers were normally distributed. The scores were then compared and subjected to Student's  $t$ -test which delivered results for interpretation.

| Cronbach's $\alpha$      | pre   | post  |
|--------------------------|-------|-------|
| organizing               | 0.859 | 0.901 |
| elaborating              | 0.808 | 0.833 |
| repeating                | 0.794 | 0.808 |
| metacognitive strategies | 0.768 | 0.822 |

Table 1: Cronbach's  $\alpha$ 

| N                   | pre | post |
|---------------------|-----|------|
| not MP <sup>2</sup> | 200 | 107  |
| SLG                 | 24  | 10   |
| SDG                 | 28  | 21   |
| MG                  | 7   | 7    |

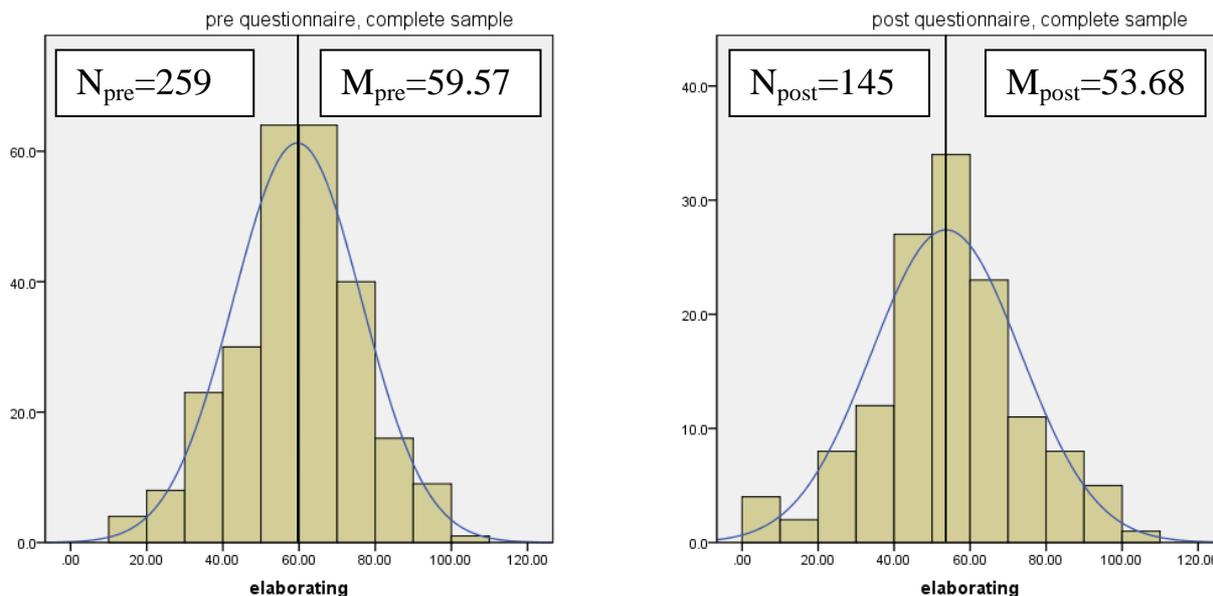
Table 2: Number of usable questionnaires

## RESULTS

All Cronbach's  $\alpha$  values were above 0.7, see table 1. As is usual in statistics, not all questionnaires sent out were filled in properly. The numbers of the usable questionnaires are given in table 2. Those with more than five empty spaces were deleted, questionnaires with up to five empty spaces were completed with the middle value 3 of the Likert scale. The low number of usable questionnaires from MG means that results from this group have to be interpreted very carefully, if at all.

| pre<br>post         | organizing                             | elaborating  | repeating  | metacognitive strategies                             |
|---------------------|--|--|--|--|
| not MP <sup>2</sup> | M=47.91; SD=20.85<br>M=46.52; SD=25.33 | M=60.02; SD=16.80<br>M=53.74; SD=19.86               | M=43.88; SD=18.24<br>M=41.22; SD=18.94               | M=54.70; SD=14.66<br>M=52.44; SD=18.29               |
| SLG                 | M=52.73; SD=22.75<br>M=53.44; SD=25.49 | M=52.60; SD=16.78<br>M=49.38; SD=13.88               | M=46.28; SD=21.13<br>M=27.50; SD=13.47               | M=57.01; SD=13.63<br>M=53.63; SD=10.62               |
| SDG                 | M=49.44; SD=19.75<br>M=41.19; SD=20.86 | M=62.16; SD=13.72<br>M=51.42; SD=15.42               | M=47.36; SD=16.25<br>M=35.71; SD=16.60               | M=58.77; SD=16.58<br>M=45.25; SD=13.69               |
| MG                  | M=47.77; SD=18.98<br>M=50.00; SD=25.52 | M=60.27; SD=26.62<br>M=62.50; SD=28.53               | M=43.37; SD=23.37<br>M=45.92; SD=24.87               | M=51.95; SD=10.12<br>M=50.32; SD=14.10               |
| complete sample     | M=48.52; SD=20.80<br>M=46.78; SD=24.88 | <b>M=59.57; SD=16.86</b><br><b>M=53.68; SD=19.66</b> | <b>M=44.46; SD=18.39</b><br><b>M=40.00; SD=18.98</b> | <b>M=55.28; SD=14.69</b><br><b>M=51.83; SD=17.30</b> |

Table 3: Statistics for (meta)cognitive learning strategies, distinguished by MP<sup>2</sup> groups



The Kolmogorov-Smirnov tests for normal distribution confirmed that the data was normally distributed. As an example the histograms for *elaborating* are given above, with their approximate normal distribution curves.

The most interesting part of our evaluation is doubtless the comparison of the scores of the pre and post questionnaires. Our data allows us to compare not only the pre and post questionnaires of the entirety, but also those of the three groups within MP<sup>2</sup>. As expected, not all of these comparisons are statistically significant; therefore we will concentrate on those which are. However, table 3 gives a comprehensive overview. It is obvious that the scores for not MP<sup>2</sup> participants and the complete sample hardly differ. This is mainly due to the numbers, see table 2 above.

Student's t-tests provided statistically reliable results about in which interval the scores differed with a certain probability. For *elaborating* (see histograms above), *repeating* and *metacognitive learning strategies* the tests showed that the lower scores in the post questionnaires (boldface in table 3) for the complete sample were significant, see table 4.

|                          | Levene significance | variances | confidence interval (p=.95) |
|--------------------------|---------------------|-----------|-----------------------------|
| elaborating              | .106                | ≠         | [-9.81; -1.97]              |
| repeating                | .554                | =         | [-8.34; -.58]               |
| metacognitive strategies | .289                | ≠         | [-6.89; -.01]               |

Table 4: Negative confidence intervals produced by Student's t-test

Taking into account that this is what the students said about themselves, there are several possible interpretations. One of them is that while the students considered their learning strategies (with respect to *elaborating*, *repeating* and *metacognitive strategies*) more or less adequate at the beginning of the semester, at the end of the

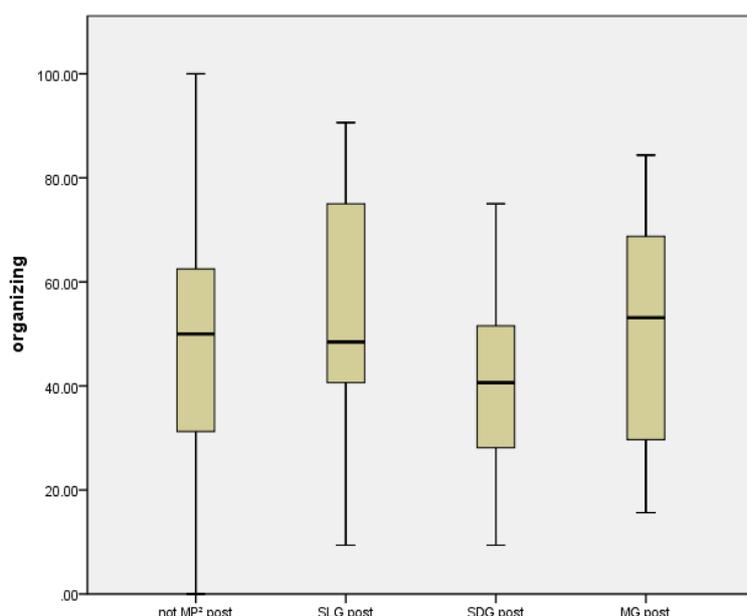
semester they realized they had not worked hard enough. This interpretation makes sense if we take into account that university mathematics differs considerably from school mathematics (cf. Zucker, 1996; Engelbrecht, 2008). Another possible interpretation is that though the students intended to work seriously, they did not do so, for whatever reason. An answer to the problem which of these interpretations is correct may be found in the interviews done with some of the students. Those will be evaluated in the future; at first sight there is no indication that the students met learning obstacles other than inadequate learning strategies and the advanced level of university mathematics.

A more detailed evaluation is to follow, in this paper the focus is on three significant differences between the MP<sup>2</sup> groups, see table 5.

|  | Levene significance | variances | confidence interval (p=.95) |
|--|---------------------|-----------|-----------------------------|
| <i>organizing</i><br>SLG / SDG post                | .457                | ≠         | [-9.28; 33.77]              |
| <i>repeating</i><br>SLG / not MP <sup>2</sup> post | .445                | ≠         | [-23.77; -3.67]             |
| <i>metacognitive strategies</i><br>SLG / SDG post  | .277                | ≠         | [-2.77; 19.54]              |

Table 5: Confidence intervals of mainly identical algebraic sign, produced by Student's t-test

The first is the very much higher score in *organizing* of the “Supported Learning Group” (SLG) than of the “Self-Directed Group” (SDG) at the end of the semester ( $M=53.44$ ;  $SD=25.49$  to  $M=41.19$ ;  $SD=20.86$ ), see boxplot diagrams for more detailed information on the distribution, its median and quartiles. The main difference



between these two groups is the fact that SLG met with a tutor once a week whereas SDG did not. Both groups had access to an e-learning course and filled in a learning diary. This scenario means the results can be regarded as due to the tutorials aimed at advancing learning strategies. The figures prove that the tutorial resulted in a significantly higher score in the learning strategy *organizing*, so the *Math/Plus* concept can be seen as a success in this respect.

The second result to be looked

upon here is the very much lower score in *repeating* of SLG than of students not participating in MP<sup>2</sup> at the end of the semester ( $M=27.50$ ;  $SD=13.47$  to  $M=41.22$ ;  $SD=18.94$ ). It may be that the MP<sup>2</sup> interventions resulted in a focus on more advanced learning strategies, somewhat neglecting *repeating*. It may also be that in retrospect the students of SLG thought their repeating strategies were inadequate when compared to what would have been necessary. Be that as it may, in this respect the participants in the central *Math/Plus* group scored disappointingly. This result will be taken as a hint how to improve *Math/Plus* in the future.

The third and last difference in scores is the higher score in *metacognitive strategies* of SLG than of SDG at the end of the semester ( $M=53.63$ ;  $SD=10.62$  to  $M=45.25$ ;  $SD=13.69$ ). As metacognition is the first step on the way to modifying and regulating behavior, this result means that the personal tutoring that SLG experienced is a substantial aspect. Looking at the figures for the students not taking part in MP<sup>2</sup> which are similar to those of SLG, it may be concluded that an e-learning course and a learning diary *without* the support of a weekly tutorial and the personal contacts that go with it can result in increased frustration that finds its expression in the comparatively low score in *metacognitive strategies*. It must therefore be considered carefully if students are to be exposed to interventions without offering personal support.

## **CONCLUSIONS - RELEVANCE FOR THE TEACHING OF MATHEMATICS**

Traditionally, mathematics is regarded as a difficult subject; therefore, the focus of teaching mathematics often lies on the subject matter. Recently the attention has shifted to learning strategies (Rach & Heinze, 2011) which makes sense as they are the universal tool when tackling new areas. In particular, the transition from school to university mathematics requires a different and more elaborated learning behavior than that generally promoted in schools. Reaching a higher level of expertise in mathematics needs knowledge and command of one's own cognitive processes.

Our results show that working on learning strategies in combination with personal and digital support systems can advance the cognitive strategies notably. It remains to be examined which interventions are the most effective; however, the personal component seems to be of importance.

## References

- Dehling, H., Glasmachers, E., Härterich, J., & Hellermann, K. (2010). MP<sup>2</sup> - Mathe/Plus/Praxis: Neue Ideen für die Servicelehre. *Mitteilungen der Deutschen Mathematiker-Vereinigung*, 18, 252.
- Dubinsky, E., & McDonald, M. A. (2001). APOS theory: A constructivist theory of learning in undergraduate mathematics education research. In D. Holton (Ed.), *The Teaching and Learning of Mathematics at University Level: An ICMI Study* (pp. 273-280). Dordrecht, Netherlands: Kluwer Academic Publishers.
- Duffin, J. M., & Simpson, A. (1993). Natural, conflicting and alien. *Journal of Mathematical Behavior*, 12, 313-328.
- Engelbrecht, J. (2008). Adding structure to the transition process to advanced mathematical activity. In D. de Bock, B. D. Søndergaard, B. A. Gómez, & C. C. L. Cheng (Eds.), *Proceedings of ICME 11*. (pp. x-y). Monterrey, Mexico: ??.
- Flavell, J. H. (1979). Metacognition and cognitive monitoring: A new area of cognitive-developmental inquiry. *American Psychologist* 34 (10), 906-911.
- Gray, E., & Tall, D. (1991). Duality, ambiguity and flexibility in successful mathematical thinking. In F. Furinghetti (Ed.), *Proceedings of the 15th International Conference for the Psychology of Mathematics Education, Vol. 2* (pp. 72-79). Genoa, Italy: University of Genoa.
- de Guzmán, M., Hodgson, B. R., Robert, A., & Villani, V. (1998). Difficulties in the passage from secondary to tertiary education. In G. Fischer & U. Rehmann (Eds.), *Proceedings of the International Congress of Mathematicians – Documenta Mathematica Extra Volume III*, 747-762.
- Hoyles, C., Newman, K., & Noss, R. (2001). Changing patterns of transition from school to university mathematics. *International Journal of Mathematics Education in Science and Technology* 32(6), 829-845.
- Landmann, M., & Schmitz, B. (2007). Die Kombination von Trainings mit standardisierten Tagebüchern: Angeleitete Selbstbeobachtung als Möglichkeit der Unterstützung von Trainingsmaßnahmen. In M. Landmann & B. Schmitz (Eds.), *Selbstregulation erfolgreich fördern. Praxisnahe Trainingsprogramme für effektives Lernen* (pp. 151-163). Stuttgart, Germany: Kohlhammer.
- Liston, N., & O'Donoghue, J. (2008). The influence of affective variables on students' transition to university mathematics. *Topic Study Group 30, ICME 2008*. Retrieved from <http://tsg.icme11.org/tsg/show/31>
- Marais, C. (2000). Comparing first years' reasoning styles in formal logic and other mathematical contexts. Masters dissertation, University of Warwick, UK (published?).
- Piaget, J. (1973). *Die Entwicklung der elementaren logischen Strukturen*. Düsseldorf, Germany: Schwann.
- Rach, S., & Heinze, A. (2011). Studying mathematics at university: The influence of learning strategies. In B. Ubuz (Ed.) *Proceedings of the 35<sup>th</sup> Conference of the*

*International Group for the Psychology of Mathematics Education, Vol. 4* (pp. 9-16). Ankara, Turkey: PME.

Rach, S., & Klostermann, M. (2011). Die Studieneingangsphase in den Fächern Chemie und Mathematik. *IPN Blätter 2/11*, 1-3.

Schmitz, B., & Wiese, B. S. (2006). New perspectives for the evaluation of training sessions in self-regulated learning: Time-series analysis of diary data. *Contemporary Educational Psychology*, 31, 64-96.

Tall, D. O. (1997). *Advanced mathematical thinking*. London, England: Kluwer Academic Publishers.

Tall, D. O. (2004). Building theories: The three worlds of mathematics. *For the Learning of Mathematics*, 24 (1), 29–33.

Wild, K.-P., & Schiefele, U. (1994). Lernstrategien im Studium. Ergebnisse zur Faktorenstruktur und Reliabilität eines neuen Fragebogens. *Zeitschrift für Differentielle und Diagnostische Psychologie*, 15, 185-200.

Zucker, S. (1996). Teaching at the university level. *Notices of the American Mathematical Society* 43(8), 863-865.

## APPENDIX (LIST QUESTIONS)

### 1 Organizing:

- 1.1 I make charts, diagrams and graphics in order to have the subject matter in front of me in a structured form.
- 1.2 I compile short summaries of the most important contents as a mnemonic aid.
- 1.3 I go over my notes and structure the most important points.
- 1.4 I try to order the subject matter in a way that makes it easy for me to remember.
- 1.5 I compile a summary of the main ideas out of my notes, the script or other sources.
- 1.6 I underline the most important parts in my notes or in the texts.
- 1.7 For bigger amounts of subject matter I find an arrangement that mirrors the structure best.
- 1.8 I assemble important terms and definitions in my own lists.

### 2 Elaborating

- 2.1 I try to find connections to other subjects or courses.
- 2.2 I think of practical applications of new concepts.
- 2.3 I try to relate new terms or theories to terms or theories I already know.
- 2.4 I visualize new issues.
- 2.5 In my mind I try to connect newly learnt facts to what I already know.
- 2.6 I think of practical examples for certain curricular facts.
- 2.7 I relate what I am learning to my own experiences.
- 2.8 I wonder if the subject matter is relevant for my everyday life.

### **3 Repeating**

- 3.1 I imprint the subject matter from the lecture on my memory by repeating it.
- 3.2 I read my notes several times in a row.
- 3.3 I learn key terms by heart in order to remember important facts better in the exam.
- 3.4 I commit a self-compiled compendium to memory.
- 3.5 I read a text and try to recite it at the end of each paragraph.
- 3.6 I commit rules, technical terms or formulas to memory.
- 3.7 I learn the subject matter by heart using scripts or other notes.

### **4 Metacognition: Planning**

- 4.1 I try to consider beforehand which areas of certain topics I have to study and which I do not have to study.
- 4.2 I decide in advance how much subject matter I would like to work through in this session.
- 4.3 Before starting on an area of expertise, I reflect upon how to work most efficiently.
- 4.4 I plan in advance in which order I want to work through the subject matter.

### **5 Metacognition: Monitoring**

- 5.1 I ask myself questions on the subject matter in order to make sure that I have understood everything correctly.
- 5.2 In order to find gaps in my knowledge I sum up the most important contents without using my notes.
- 5.3 I work on additional tasks in order to determine if I have truly understood the subject matter.
- 5.4 In order to check my own understanding I explain certain parts of the subject matter to a fellow student.

### **6 Metacognition: Regulating**

- 6.1 If I am confronted with a difficult subject matter, I will adapt my learning technique to the higher demands.
- 6.2 If I do not understand everything I am reading, I will try to make a note of the gap in my knowledge and sift through the material again.
- 6.3 When an aspect seems confusing or unclear, I examine it again thoroughly.