## PREFACE

The present book is based on a course of lectures delivered by the second author at the Department of Mathematics, Indian Institute of Science, Bangalore during seven weeks in February/March 1998. The course met four hours weekly with tutorials of two hours in addition. The arrangement of chapters follows quite closely the sequence of these lectures and each chapter contains more or less the subject-matter of one week. In addition to the exercises covered in the tutorial sessions, further exercises are added at the appropriate places to enhance the understanding and to provide examples. We recommend to look at them while studying the text. To those exercises which are used at other places sufficient hints for straightforward solutions are given. Chapter 7 is an expanded version of the lectures given in the last week (and would at least need two weeks to deliver). The lecture notes [12] based on a series of lectures in 1971/72 and written by Dr. Michael Lippa constituted an important model.

The objective of the lectures was to introduce Algebraic Geometry and Commutative Algebra simultaneously and to show their interplay. This aspect was developed systematically and in full generality with all its consequences in the work of A. Grothendieck, cf. [4]. In Commutative Algebra we do not introduce and use the concept of completion. In geometry we start the language of sheaves and schemes from scratch, but we avoid sheaf cohomology completely. The Riemann–Roch theorem is formulated for arbitrary coherent sheaves on arbitrary projective curves over an arbitrary field. Its proof we reduce to the case of the projective line. Instead of (first) cohomology it uses the dualizing sheaf. Since the uniqueness of this sheaf is not so important for the understanding of the Riemann–Roch theorem, its proof which uses some homological algebra is postponed to the end. We have added a lot of illustrative examples and related concepts to draw many consequences, especially about the genus of a projective curve.

We start with basic Commutative Algebra and emphasize on normalization. As geometric counterpart we then introduce the $K$-spectrum of a finitely generated algebra over a field $K$. We extend these concepts to prime spectra of arbitrary commutative rings and develop the dimension theory for arbitrary commutative Noetherian rings and their spectra. After introducing the language of sheaves we develop the theory of schemes, in particular, projective schemes. The main theorem of elimination and the mapping theorem of Chevalley are proved. Regularity, normality and smoothness are discussed in detail including the theory of Kähler differentials. We give a self-contained treatment of the module of Kähler differentials and use the sheaf of Kähler differentials as a fundamental example of a coherent and quasi-coherent module on a scheme. Before we prove the Riemann–Roch theorem we describe the coherent and quasi-coherent modules on projective schemes with the help of graded modules.

With very few exceptions full proofs are given under the assumption that the reader has some experience with the basic concepts of algebra, as groups, rings, fields, vector spaces, modules etc. It should be emphasized that, for a reader who has these prerequisites at his or her fingertips, this book is largely self-contained.

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