Commentaries and Remarks to

Patil/Storch: Introduction to Algebraic Geometry and Commutative Algebra

---

From the review by Liam O'Carroll in Mathematical Reviews [MR2648005]:

This monograph is possibly unique in offering at the graduate level an equally balanced, detailed, thoroughly integrated and relatively advanced combination of core topics in commutative algebra and algebraic geometry. The material is well-illustrated with examples. Moreover there is a wide range of exercises, many of them informative and some quite challenging. The emphasis on the algebraic side is initially on normalization, and subsequently on regular local rings and the theory of Kähler differentials. On the geometrical side, the focus is on projective curves through the perspective of sheaf theory. These themes are then mixed to good effect so as to give a treatment of the Riemann-Roch Theorem for projective curves by first establishing the result for the projective line and then using the assembled theory to prove the result in general by pulling back over a finite projection.

This approach has the double benefit of providing a springboard for more advanced theory by exhibiting in concrete detail how general machinery plays out in the restricted but important case of curves while also allowing the authors to bring together an impressively wide range of key basic topics in algebraic geometry, usually in the form of exercises.

[. . .] *, there is also a discussion of the strong (i.e. the usual) topology on an affine algebraic set whenever the base field is R or C; this allows the introduction of analytic methods so as to discuss smooth and singular points via the Jacobian criterion. In turn this leads on to a treatment of quadrics and regular double points that eventually involves topological methods and the use of Stiefel manifolds. There are also some interesting pages on planar real irreducible quadratics, cubics and curves of higher degree, complete with diagrams wonderfully labelled by their classical names.*

Next follow two chapters giving a detailed discussion of schemes and projective schemes, well illustrated by examples, which culminate in the Main Theorem of Elimination Theory and the Mapping Theorem of Chevalley on the preservation of the property of constructibility. Along the way we meet such topics as Veronese and Segre embeddings, blowing-up and resultant systems.

The penultimate chapter focuses on regular, normal and smooth points (with the Cohen-Macaulay and Gorenstein properties treated along the way), the normalization of a scheme and the module of (and eventually the sheaf of) Kähler differentials. An unusual feature here is the use of topology to discuss Harnack's equality and inequality for smooth projective real curves and to introduce the notion of genus. The tight connection between the strong and Zariski topologies for complex algebraic schemes is also gone into.

The book closes with a very rich if more demanding and advanced chapter on the Riemann-Roch Theorem for projective curves. The treatment is detailed and careful, and usefully illustrated in this context by such topics as the Riemann surface of a function field; the canonical (or dualizing) sheaf on a curve; arithmetic and geometric genera; complete intersections and the Plücker formula; Poincaré series, Hilbert-Samuel polynomials and Euler-Poincaré characteristics; multiplicity and blowing-up; the Riemann-Hurwitz formula (yielding a proof of Lüroth's Theorem), and the ramification locus and discriminant ideal; and finally a treatment of elliptic and hyperelliptic curves.

Some mild criticisms can be made. At times the reader must jump forward to find the meaning of terms. Some more advanced topics (such as faithful flatness, aspects of algebraic topology, and so on) are used without warning. Here and there the use of English is a little idiosyncratic. There are a few typographical errors. But these are very minor blemishes when compared to the wealth of material (and knowledge) on offer to the reader of this very stimulating and informative book.

From the review by Werner Kleinert in Zentralblatt der Mathematik [Zbl 1210.14001]:

[. . .] *Representing a carefully elaborated version of the course notes in book form, the current text comprises seven chapters reflecting the subject matter of the original lectures quite faithfully. Geared toward graduate students, the book only assumes some familiarity with the basic concepts of modern abstract algebra as necessary prerequisites, whereas it is utmost detailed and largely self-contained otherwise.*

As for the general approach, the authors have adopted A. Grothendieck's fundamental viewpoint and his language of modern algebraic geometry, which is heavily based on the conceptual and methodological framework of commutative algebra, thereby displaying the corresponding path of historical development in a particulary accentuated manner. [...]

The concluding Chapter 7 presents the highlight of this versatile introductory textbook: the classical Riemann-Roch Theorem for projective algebraic curves over an arbitrary ground field. In fact, the authors offer a rather general approch to this fundamental theorem in algebraic geometry in as much as they allow arbitrary curve singularities and state it for arbitrary coherent sheaves. The authors' proof of the Riemann-Roch Theorem does not involve any sheaf cohomology. Instead it is based on a fine analysis of coherent and quasi-coherent module sheaves on projective schemes by means of the theory of graded rings and modules in commutative algebra.

In the course of the entire exposition, a wealth of fundamental concepts, methods, techniques, and results from both algebraic geometry and commutative algebra is lucidly provided, with full proofs and profound explanations all through. A large number of exercises after each section, many of which come with precise hints for solution, invite the reader to get acquainted with related topies, further importent results, or additional instructive examples in the context of the core material of the book. All together, this masterly written text must be seen as an excellent introduction to modern algebraic geometry and advanced commutative algebra in their inseparable relationship.