Making maths more attractive – how real applications increase first year students' motivation

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Abstract

Too difficult, too abstract, too theoretical – many first year engineering students complain about their mathematics courses because they are neither interested, nor see they the need to cope with it. The project *MathePraxis* aims to resolve this disaffection. It links mathematical methods as they are taught in the first semesters with practical problems from engineering applications – and thereby gives first year engineering students a vivid and convincing impression of where they will need mathematics in their later working life. In this way, the project aims to enhance motivation in the initial phase of their studies and to increase the retention rate among engineering students in the long term.

We report on the implementation of *MathePraxis* at Ruhr-Universität Bochum: We have developed a course on designing a mass damper which combines basic mathematical techniques with an impressive experiment, and we have examined in an accompanying evaluation how the project influenced the students' motivation. This opens up new perspectives how to address the need for a more practically oriented mathematical education in engineering sciences.

Keywords: first year students; mathematics; motivation; practical orientation; mass damper; oscillation; differential equations

1 Fundamental problems in engineering education

1.1 Mathematics – exasperating obstacle for first year students?

Students in mechanical, civil and environmental engineering often have serious problems during their first year at university. These problems naturally arise, to some extent, from the structure of engineering education in Germany. First, there usually are no technics lessons at secondary school, and the students come to university with a wild mixture of beliefs and perceptions about their subject that often centre on the final state of being an engineer, but lack any idea about the way how to get there; in other words: they do not have any idea about the content and the academic workload of their just beginning education (Heublein et al., 2010, p. 158–160). Second, the change from secondary to tertiary education especially in mathematics is immanently hard and has been focus of much research in the last years. Beside cognitive and cultural challenges, Guzmán et al. (1998, p. 752) list didactical obstacles that students are confronted with:

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> The mathematics is different not only because the topics are different, but more to the point because of an increased depth, both with respect to the technical abilities needed to manipulate the new objects and the conceptual understanding underlying them.

In the mathematics courses at university, the students encounter not only a higher tempo and a broader range of topics, but first and foremost an abstract formal world which they do not know from school (Tall, 2004) which requires a deeper understanding (Zucker, 1996; Engelbrecht, 2010). Rach and Heinze (2011) point out that all the different kinds of challenges at secondary-tertiary transitions require specific learning strategies students may not have developed throughout their school time. Third, the typical structure of engineering studies, at least in Germany, often does not match the students' dispositions and expectations: they usually have a strong interest in technics, building, constructing and machines, but the initial semesters consist in large part of courses in mathematics and mechanics which arise from the didactical concept that students need a certain basic knowledge before they will be able to treat real problems. Unfortunately, a large number of students struggles with this framework: over the months, they lose their interest, because they do not see the need for learning the abstract and difficult methods.

Indeed, the three-year study by Seymour and Hewitt (1997) confirms that it is not something like the size of classes, the poor quality of teaching assistants or the students' personal incompetence which causes the attrition, but rather the curriculum design. Tobias (1990) states that didactical and conceptual deficiencies are the reason why students assess the subject engineering as hard: the first semesters are lacking explanations in an engineering context, and the students cannot apply what they have to learn before the advanced study period – but some of them drop their studies before that point.

1.2 The *MathePraxis* approach

In Germany, alarming 34% of all first-year students in mechanical engineering give up their subject (Heublein et al., 2010). More than the half of them report that wrong expectations towards the study, a decline in interest or the desire to work practically were the reason (Minks, 2000). In order to counter this drop-out rate, we developed the project *MathePraxis*, in which first year students can discover the connection between the mathematical methods taught in their courses and real engineering applications. There obviously is a lack of such kind of motivating insight in the mathematical education.

MathePraxis is a project based at Ruhr-Universität Bochum in Germany which offers support for first year students in the degree programmes mechanical engineering, civil engineering and environmental engineering by an additional course in which the students have to solve an application-orientated problem. *MathePraxis* was implemented at Ruhr-Universität Bochum in 2011 and supported by Stifterverband für die Deutsche Wissenschaft and Heinz Nixdorf foundation. For details on the larger context see Dehling et al. (2010), for details on the accompanying project *MathePlus* which concentrated on the above mentioned problems concerning learning strategies, cognitive dispositions and beliefs in the first semester, see Glasmachers et al. (2011) and Griese et al. (2011a,b).

Since usual degree programmes do not leave much time for extra projects, we did not focus on modelling or on general problem-solving skills, but we adapted problems from real applications, which were suggested by engineers from the mechanical engineering department, to the level of the first year mathematics courses (which cover basic linear algebra, geometry, calculus of one or many variables and differential equations). In the resulting projects, an interesting real problem is given to the participants, and they have to solve it during the semester. In the first run of *MathePraxis* in summer 2011, we realised three such projects which dealt with the control of a Segway, a sway control system for cranes and the effective CPU cooling by a ribbed cooler. Even though it is in the nature of real problems that they are unclear and complex and require advanced methods, time, technical effort and experience which all is not at hand for first year students, we could show that it is indeed possible to link real applications with basic mathematics successfully (Härterich et al., 2012). We believe that this kind of appealing insight boosts unsupervised, independent learning and increases motivation (Rooch et al., 2012). Classification and comparisons to other approaches can also be found in these articles.

In this paper, we present a new and improved project in the second run of *MathePraxis* in 2012 and evaluate its effect. The paper is organized as follows: In Section 2, we describe the design of our course; in Sections 3 and 4, we present the mathematical and technical content and the experimental setup; Section 5 is devoted to the accompanying evaluation of the project.

2 Combining mathematics, applications and experiments

In 2012, we repeated the project on the control of a Segway transporter and the project on a sway control system for cranes from 2011, see Härterich et al. (2012), and we developed a new project about designing a mass damper. After the first run of *MathePraxis*, many participants reported that they liked the project, but although it was much more problem-orientated than any other course, they had desired to do an experiment in order to check their mathematical findings in reality. So for the purpose of letting the students truly experience the meaning and the effect of their calculations, we developed the mass damper project with an experimental setup. This is described in Section 3 and 4.

2.1 Recruitment of participants

All students in the regular mathematics course *Mathematics II for Engineering, Civil Engineering and Environmental Engineering* were invited to participate in *MathePraxis*. The project was introduced by a short presentation and advertised by posters all around the campus. As in 2011, we had capacities to accept 30 students. Applicants had to have passed the final exam of the mathematics course *Mathematics I for Mechanical Engineering, Civil Engineering and Environmental Engineering* and they had to write a short letter of motivation. Because the project means additional workload and the academic and intellectual benefits alone may not seem sufficient profit to everyone, all participants obtained 2 or 3 credit points for *MathePraxis* (depending on their field of study) for a module in their later degree programme.

We received 58 applications which demonstrates not only the students' interest in such an offer, but also the success of the first run of *MathePraxis* and the related project *MathePlus*. This impression is supported by the wording of the letters of motivation, of which we present some examples:

- Math and practice are actually two words for me which are not connected in any way.
- It would be interesting to see that the subject matter has a meaning and that its purpose is not only to distress students.
- I often wonder what I am actually calculating. Handling abstract formulas is very difficult for me when I do not see the application.
- Help!! I must urgently retrieve the fun I that had with mathematics at school and in the first semester (and I want). To be honest, I hope to get better marks in the final exam when I participate in the project.
- I am so curious about this project that I am not listening to the professor at the moment, but writing this text.
- Like almost all other students I am wondering what the later use of mathematics is and how I can apply the matter in practice.
- Researching, trying something out and practical applications are not offered enough in the first semesters which takes away the fun.

- In physics, I love it to solve real problems, and it would be great if I could do this in mathematics as well.

We chose 36 students to participate in the six groups of *MathePraxis* in such a way that in each group there were male and female students from all three degree programmes and from all proficiency levels (as indicated by their result in the final exam of *Mathematics I*). Three students left the project because they wanted to concentrate on the regular courses or they did not like the offered date, such that 33 students took part in the run of 2012. Two groups of 4–6 students dealt independently each with the Segway, the crane control and the mass damper.

2.2 Structure of the project

At the beginning, the application-orientated problem was presented in each group. In the new project, this was the question: *How can a swinging in a building, for example at an earthquake, be damped?* It is clear that a trial and error approach is not successful here, because it may take a lot of buildings until one finds the correct way – if at all. On the basis of a simple model, this question should be solved by the students during the semester.

The groups had to work mainly on their own; but we offered a weekly meeting of one hour for each group where students could ask questions and discuss concepts and their results and where we guided them towards the right direction since we did not had the time to let the (young and unexperienced) students model on their own (as mentioned, *MathePraxis* was an additional course beside the regular schedule). In order to provide a guideline, we gave weekly assignments to the students, for example:

- Determine the equation of movement $x(t)$ of the mass-spring system when it is excited by the electric motor.
  - First, calculate the force which displaces the upper end of the spring, i.e. which excites the pendulum.
  - Take this force into account in the balance of forces and determine the (inhomogeneous) differential equation which describes the movement of the mass-spring system.
  - Give its general solution. To this end, find a general solution $x_h(t)$ and one particular solution $x_p(t)$.
- Read up on all terms that occur in the process, e.g. spring constant, equation of movement, differential equation, ... Which is the correct unit (if it has one)? What does it mean? Where does it occur in everyday life?

In the crane project, we also gave the students a short self-written textbook in which solutions were lacking – in order not to loose the overview (because the calculations can be some kind of confusing here).

By this step-by-step guidance, the students could solve the initial question within ten weeks. Moreover, in order to review work, obstacles and solutions, the students had to present their results to fellow students and interested nonspecialists in a public talk at the end of the semester. As preparation, the students received a few hours of training in presentation techniques from an external expert.

3 Mathematical content

Oscillations are in general a very important topic both in mechanical and civil engineering, see e.g. Andronov et al. (1966), Lin (2000), Lin (2005) or Ricciardelli, Pizzimenti and Mattei (2003). Although there exist several applications where oscillations are desired or even demanded (grocery industries, lab equipment), they can transform to great error wells which originate failures of engineering structures. Generator shafts, helicopter blades, skyscrapers and bridges are examples where oscillations have strictly to be under control – which means in this context on the one hand the knowledge about the resonant or eigen frequencies and on the other hand the redemption of oscillations which may be carried out in an active (helicopter blade) or passive way (skyscraper). Periodic external loads whose frequency equals the eigen frequency of the system force the structure to oscillate with amplitudes that grow over all limits if there is only little damping. But in any case, high amplitudes may provoke large strains and thus stresses. If stresses reach critical limits, the structure collapses (Billah and Scanlan, 1991).

Periodic forces are not as academic as it could be assumed at first glance: shafts will always exhibit eccentricities so that the rotating shaft stimulates itself; winds or earthquakes are in many cases like waves with more or less constant periodic length. Hence, a general comprehension of oscillating systems is one of the main goals in teaching mechanical engineering.

It turns out that the eigen frequencies correspond to the eigen values of a certain matrix (the stiffness matrix normalized by the entries of the inertia matrix of the system's differential equation of motion). The eigen modes can be identified as the eigen vectors. Thus, the analysis of oscillations is predestinated to combine teaching of mathematics and mechanics. From the mechanical side, the derivation of the system's differential system of equations has to be carried out, whereas from the mathematical side, solution techniques are needed.

Since our project is dedicated to students from the second semester, and, as we pointed out, it is additional to the regular courses which already take a lot of time, we focus on *passive redemption.* Skyscrapers which are built in regions vulnerable to earthquakes are often equipped with massive weights somewhere in the building (not in the grounding). These weights are supported in a way that allows movement (swinging). The intention of the construction is to shift the eigen frequencies of the skyscraper to a level which is less likely to be close to that in which earthquakes may occur. Such a damping by weights transforms the movement energy which is provided to the skyscraper by the earthquake into (nonhazardous) heat energy. This allows a moving skyscraper which will not collapse.

We take this example to build our project's idea: The final goal is the redemption of a *mass-spring system* on which a periodic force acts in eigen frequency of the system.

To this end, the mass-spring system is expanded by a second mass-spring system which is fixed to the mass of the first system. The constants of the spring can be measured by the students (simply by noting the displacement of the respective spring if a certain known mass is connected to the spring). The mass of the second system has then to be chosen in a way that the movement of the first mass is entirely erased when the mentioned periodic force in eigen frequency of the one-mass-spring system is applied to the two-mass-spring system. In order to find the desired mass for redemption, first the mechanical movement equations for both systems have to be found for which the well known Newton's laws have to be applied. By our experimental setup, see Section 4, a very comprehensive environment is created which supports the students in deriving and using the law of linear momentum given by

$$\sum_i F_i = m\ddot{x} \tag{1}$$

where F_i are all forces acting on the body with mass m and where the acceleration is expressed by the second time derivative of the spatial coordinates, denoted by $\ddot{x}$.

3.1 One-mass-spring system

First, we derive the equation of movement for the one-mass-spring system, see Figure 1 (left). Application of Newton's law, as given in (1), yields

$$-F_1 = m_1\ddot{x}_1, \tag{2}$$

where m_1 is the upper mass and x_1 is its vertical coordinate with positive direction downwards.

First, we want to calculate the eigen frequency. We have $F_1 = c_1x_1$, where c_1 is the measured spring constant and obtain the homogeneous equation of motion $\ddot{x}_1 + \omega^2 x_1 = 0$ with $\omega^2 = c_1/m_1$

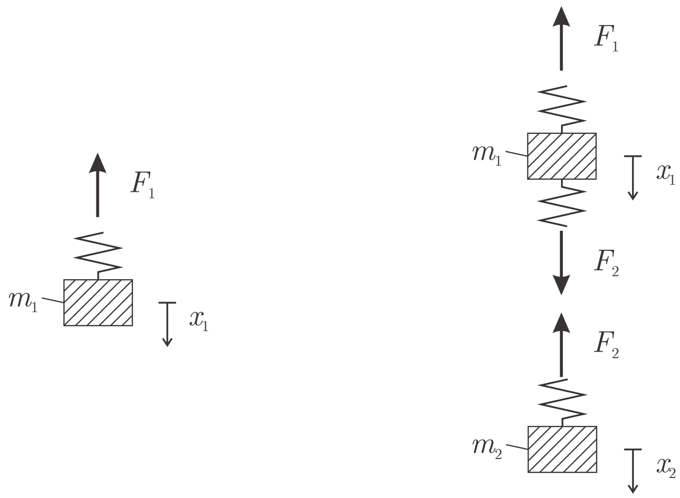

Figure 1: Left: Liberated one-mass-spring system with spring force F_1 and positive coordinate x_1, pointing downwards. Right: Liberated two-mass-spring system with additional spring force F_2 and positive coordinate x_2, pointing downwards.

which is solved by applying the ansatz $x_1(t) = \hat{x}_1 e^{\lambda t}$ with the amplitude $\hat{x}_1$. It yields the well known final solution

$$x_{1,h}(t) = a_1 \cos(\omega t) + b_1 \sin(\omega t) \tag{3}$$

where a_1 and b_1 have to be found from the initial conditions of position and velocity. Here, we draw the student's attention to the identity $e^{i\omega t} = \cos(\omega t) + i \sin(\omega t)$ where this knowledge from the mathematics lesson is in great accordance to the empirical expectation that the oscillation should be of an "harmonic type".

The mass m_1 is connected to the support through the spring. The other end of the spring can be moved periodically. In this case, the spring force reads $F_1 = c_1(x_1 - u)$ where u is the time dependent periodic displacement of the upper end of the spring. It is given as $u = r\sin(\Omega t)$ where r is the radius of the wheel at which the connection rod is fixed, Ω is the frequency of the wheel and t denotes the time, see Figure 2. Then, the equation of motion is

$$\ddot{x}_1 + \omega^2 x_1 = \frac{c_1}{m_1} u = C_1 \sin(\Omega t) \tag{4}$$

with $C_1 = c_1 r/m_1$ which is an inhomogeneous differential equation of second order. The solution for (4) is a combination of homogeneous and particular solution, $x_1 = x_{1,h} + x_{1,p}$. For the particular solution we follow the usual way of an *ansatz of the right hand side* which is $x_{1,p} = d_1 \sin(\Omega t) + d_2 \cos(\Omega t)$. Inserting this into (4), a comparison of the coefficients yields $x_{1,p} = C_1/(\omega^2 - \Omega^2)\sin(\Omega t)$. From this, we find easily that the critical external frequency or eigen frequency is $\Omega^\star = \omega$.

3.2 Two-mass-spring system

The system consisting of one mass and one spring is now expanded by a second mass-spring system which is fixed at the first mass. The spring constant of the second spring is denoted as c_2, the second mass as m_2 and the displacement relative to the statical elongation of the second spring as x_2, also pointing downwards. The first spring force remains the same as before. The force for the second spring is $F_2 = c_2(x_2 - x_1)$. Analogously to the one-mass-spring system, both masses are liberated and the interaction between the masses is included by the respective forces, see Figure 1 (right). Application of Newton's law to both masses yields for the eigen oscillation without any external force

$$m_1\ddot{x}_1 + (c_1 + c_2)x_1 - c_2x_2 = 0 \tag{5}$$
$$m_2\ddot{x}_2 - c_2x_1 + c_2x_2 = 0 \tag{6}$$

This can be written in vectorial form as

$$\boldsymbol{M} \cdot \ddot{\boldsymbol{x}} + \boldsymbol{C} \cdot \boldsymbol{x} = \boldsymbol{0} \tag{7}$$

with mass matrix $\boldsymbol{M}$, stiffness matrix $\boldsymbol{C}$ and displacement vector $\boldsymbol{x}$:

$$\boldsymbol{M} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}, \qquad \boldsymbol{C} = \begin{pmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{pmatrix}, \qquad \boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{8}$$

If the system is moved by the wheel, the right hand side of (7) turns into $\boldsymbol{F} = (c_1u,\, 0)^T$, compare to Section 3.1. It is possible to reformulate the system of equations of motion given in (5) and (6) by dividing the first equation by m_1 and the second by m_2. Then (7) reads

$$\ddot{\boldsymbol{x}} + \boldsymbol{A} \cdot \boldsymbol{x} = \boldsymbol{0} \tag{9}$$

with

$$\boldsymbol{A} = \begin{pmatrix} \frac{c_1+c_2}{m_1} & -\frac{c_2}{m_1} \\ -\frac{c_2}{m_2} & \frac{c_2}{m_2} \end{pmatrix}. \tag{10}$$

In order to solve (9), we pursue the same way as in the previous section and use the ansatz $\boldsymbol{x}(t) = \hat{\boldsymbol{x}}e^{\lambda t}$ with the vector of amplitudes $\hat{\boldsymbol{x}} = (\hat{x}_1,\, \hat{x}_2)^T$. Inserting this into (9) yields

$$\lambda^2\hat{\boldsymbol{x}} = -\boldsymbol{A} \cdot \hat{\boldsymbol{x}}\,. \tag{11}$$

From (11) it is obvious that the resulting mechanical problem is identical to an eigensystem problem with the eigenvalues λ and the eigenvectors $\hat{\boldsymbol{x}}$. The eigenvalues are found from the roots of the characteristic polynomial $\det(\lambda^2\boldsymbol{I} - \boldsymbol{A})$. For the particular solution of the moving system, $\boldsymbol{M} \cdot \ddot{\boldsymbol{x}} + \boldsymbol{C} \cdot \boldsymbol{x} = \boldsymbol{F}$, we choose again an ansatz of the right hand side, $\boldsymbol{x}(t) = \hat{\boldsymbol{x}}\sin(\Omega t)$, since $u = r\sin(\Omega t)$, see Section 3.1. Inserting this gives

$$(-\Omega^2\boldsymbol{M} + \boldsymbol{C}) \cdot \hat{\boldsymbol{x}} = \boldsymbol{F} \tag{12}$$

which can be solved for $\hat{\boldsymbol{x}}$, e.g. by Cramer's rule. The solution is

$$\hat{x}_1 = \frac{\bar{\omega}_{10}^2 r(\bar{\omega}_2^2 - \Omega^2)}{(\bar{\omega}_1^2 - \Omega^2)(\bar{\omega}_2^2 - \Omega^2) - \bar{\mu}\bar{\omega}_2^4} \tag{13}$$
$$\hat{x}_2 = \frac{\bar{\omega}_{10}^2 r\bar{\omega}_2^2}{(\bar{\omega}_1^2 - \Omega^2)(\bar{\omega}_2^2 - \Omega^2) - \bar{\mu}\bar{\omega}_2^4} \tag{14}$$

with $\bar{\omega}_{10}^2 := c_1/m_1$, $\bar{\omega}_1^2 := (c_1 + c_2)/m_1$, $\bar{\omega}_2^2 := c_2/m_2$ and $\bar{\mu} := m_2/m_1$.

Similar as for the one-mass-spring system, the students see that a problem for the two-mass-spring system arises when the denominator of $\hat{x}_1$ or $\hat{x}_2$ becomes zero. This occurs if

$$\Omega^\star_{1,2,3,4} = \pm\sqrt{\frac{\bar{\omega}_1^2 + \bar{\omega}_2^2}{2} \pm \sqrt{\frac{\bar{\omega}_1^2 - \bar{\omega}_2^2}{4} + \bar{\mu}\bar{\omega}_2^2}}\,. \tag{15}$$

Thus, $\Omega^\star_{1,2,3,4}$ are the (double) roots of the denominator in (12). If Ω equals one of the $\Omega^\star_i$, a resonance disaster occurs since the amplitudes of the masses grow over all limits. From (15), the students see that, depending on the system's number of degrees of freedom, an identical number of eigen frequencies results.

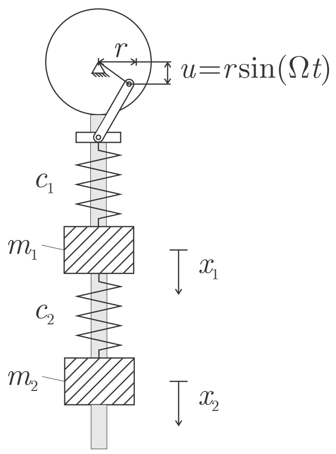

Figure 2: Idealized plot of our experimental setup.

The aim of the project is the redemption of the movement of the first mass, thus we demand $\hat{x}_1 = 0$. Using this and inserting the solution, (13) and (14), into (6) yields

$$-m_2\Omega^2\hat{x}_2\sin(\Omega t) + c_2\hat{x}_2\sin(\Omega t) = 0 \tag{16}$$

from which we find

$$m_2 = c_2/\Omega^2 \tag{17}$$

for the redemption of mass m_1. This holds for every frequency of movement, so for $\Omega = \Omega^\star$.

4 Experimental setup

The topics oscillations and damping, as presented in Setcion 3, are usually a regular part of basic engineering education. However, the interrelation between mechanics and mathematics is rarely taken into account in courses and lectures such that the students possess some knowledge about eigenvalues and eigenvectors, but a demonstration of their meaning, i.e. any physical compliance is missing. The aim of this project was to close this gap. Hence, a demonstrator was build up capable of exhibiting the aforementioned oscillations. The students had to calculate the appropriate mass m_2 such that the oscillating of the first mass m_1 is extincted, and with this demonstrator, the students could not only see the meaning of parameters like frequency and mass and a resonance disaster with their own eyes, but they could also check their results in practice.

The basis of the demonstrator is a frame formed by four metallic rods at the edges and massive wooden plates at bottom and top. In the even center of the frame, a fifth metallic rod is located which serves as bounding support for the masses. The masses themselves are ring segments so that they can easily be added or removed from the construction. Each ring segment obtains threads which allow to modify the final masses. Furthermore, the threads serve as connection interface between the masses and the springs which are winded around the rod. At the top plate an electric motor is located. It is engineered such that the resultant torque and frequency allow oscillations which exhibit eigenmodes and it is joined to a wheel which transforms the rotation movement (of the engine) to a translational movement (of the masses) by a piston rod. The radius where the piston rod is connected to the wheel is much smaller than the length of the piston rod. Hence, all changes of angles are quite small for which it holds $u = r\sin(\Omega t)$. The other end of the piston rod is connected to the upper end of the first spring so that a periodic movement is applied to the system.

The frequency of the engine can controlled only indirectly by regulating the connected voltage generator. Hence, the wheel is performed in a way that it has 90 trunnions uniformly distributed

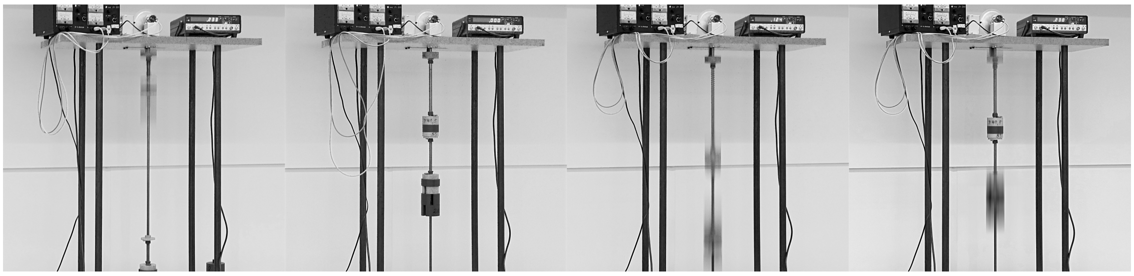

Figure 3: Our experimental setup. The one-mass-systems oscillates in its eigenfrequency (1), a second mass can be attached via threads (2), the two-mass-system oscillates in a frequency close to its eigen frequency (3), the two-mass-system oscillates in the eigen frequency of the one-mass system, where the second mass has been choosen such that it damps the movement of the first mass (4)

around its edge. A light bridge is then interrupted 90 times per turn which is measured by a frequency counter. This allows a convenient control of the applied frequency Ω.

We have built the demonstrator robust and large enough such that on the one hand it is capable of showing impressively an excitement in eigen frequency, a resonance disaster, at least for a few seconds, and on the other hand it can be used in lectures.

One very important side effect of the demonstrator was that it gave reason for discussions about the deeper meaning of a calculated result: it is not only useful to impart understanding what the mechanical counterpart of an eigen value and what an equation of motion is, but with an experimental setup like this, the students see that the solution of an equation of motion is not automatically the description of a real movement but subject to model assumptions. For example, solution (3) states that the mass will move periodically without end, and obviously, this does not describe the reality – because we did not take friction into account. Thus, outgoing from the observed movement of the pendulum, we could discuss the model with the students and spend some time on a more realistic modelling including friction which allowed us to forecast the movement in a more realistic way which was based on a bunch of algebraic transformations and the resulting amplification function as a function of the specific (dimension-free) excitation and friction. This approach gave the students a genuine and realistic impression of how mathematics is used in real life; nevertheless, to keep things practicable we rejected the approach for the further project.

5 Evaluation

Previous research has shown that the *MathePraxis* approach is feasible and effective: it is possible to link real engineering applications with basic mathematics (Härterich et al., 2012), and such a problem-oriented project indeed improves the students' attitiude towards the subject mathematics (Rooch et al., 2012). Now we want to examine the effect of *MathePraxis* on the students' motivation and their attitude towards mathematics in more detail. To this end, we have carried out a study with students who took part in *MathePraxis* and, as a control group, some who did not. In this Section, we will describe the study and discuss the results.

5.1 Background and methodology

The question of interest is: *How do real applications influence the engineering students' motivation to deal with mathematics?* Gómez-Chacón and Haines (2008) have studied a quite similar question – how the use of computers in undergraduate courses influences the students' attitude towards mathematics –, and since their questionnaire has been used in several studies from 1998 up to date, we decided to adapt it. For details about some of these studies and a didactic discussion of the attitude concept which is at the basis of the questionnaire, see the articles of Gómez-Chacón and Haines (2008) and Galbraith and Haines (1998, 2000).

The questionnaire covers three aspects of attitude:

- *Mathematics confidence* (Do students feel good about the subject mathematics? Do they expect to get good marks? Are they worried about difficult topics?)
- *Mathematics motivation* (Do the students spend time on problems? Do they like the subject mathematics? Is mathematics an appealing challenge for them?)
- *Mathematics engagement* (Do the students prefer to learn through examples rather than through memorizing? Do they check their understanding through excercises? Do they review their work?)

For our purposes, we slightly adapted the questionnaire by Gómez-Chacón and Haines (2008): in the course of translating it to German language, we changed some wording when it seemed to be more appropriate, and we left out three questions (which did not prove to carry information in the factor analysis during the score). The used questionnaire is given in Table 1.

A five-point Likert scale was applied to all items, consisting of the Likert items *strongly disagree (1), disagree (2), neither agree nor disagree (3), agree (4)* and *strongly agree (5).* To support intuitive interpretation, we transformed the Likert scale by the linear mapping $t(x) = 2.5\cdot(10x-10)$ to a range of 0 to 100.

The questionnaire was given out to all participants of *MathePraxis* and to all other students in the regular mathematics course *Mathematics II for Engineering, Civil Engineering and Environmental Engineering* who did not participate, each at the beginning (pre) and at the end of the semester (post). The evaluation was anonymous, but an anonymized identification code was assigned to each student so individual responses in the survey can be tracked. We received a total number $n_p = 19$ of usable (paired) questionnaires from the participants and a total number $n_c = 43$ of usable (paired) questionnaires from the control group.

5.2 Score

Since the original questionnaire, which has been used for several studies and whose reliability has thus been tested many times, underwent some changes, we have to assess the validity and reliability of our modified questionnaire at first. It is reasonable to do this on the largest data set possible. In fact, we have received in the participants' group a total number of 38 questionnaires from the pre evaluation and 26 from the post evaluation, and in the control group 83 and 88 questionnaires. Based on these data, we will check the validity and reliability of the modified questionnaire, but for the further analysis, we will restrict ourselves to the $n_p = 19$ and $n_c = 43$ paired questionnaires.

We have performed a confirmatory factor analysis (CFA) in order to estimate the internal consistency of the three scales, i.e. how well the items in each scale match up; the resulting loadings are given in Table 1. For comparison, we also give the loadings of the items if the analysis is based only on the $n_p = 19$ and $n_c = 43$ paired questionnaires. One cannot spot any dramatic difference.

Furthermore, we have calculated Cronbach's α for each scale in the pre and the post evaluation. It is a common rule of thumb to rate the internal consistency as good if $alpha \geq 0.8$ and as unacceptable if $\alpha < 0.5$; the results are given in Table 2. We clearly see that while there is no consistency in the third factor *engagement*, such that we will leave it out in the following considerations, the first two factors *confidence* and *motivation* are reliable. For comparison, we also give the values of Cronbach's α which are based only on the $n_p = 19$ and $n_c = 43$ paired questionnaires; we see that we can draw the same conclusions from this.

Let $X_{i,j}^{(\text{pre})}$ denote the answer of person i on question j in the first survey, $X_{i,j}^{(\text{post})}$ the answer in the second survey. Note that $1 \leq i \leq n_p$ for a person taking part in the project, $1 \leq i \leq n_c$ for

Item	*Pos*	*Load 1*	*Load 2*
Mathematics Confidence			
Mathematics is a subject in which I get value for effort.	3	0.62	0.73
The prospect of having to learn new mathematics makes me nervous.	6	0.69	0.72
I can get good results in mathematics.	24	0.78	0.79
I am more afraid of mathematics than of any other subject.	12	0.68	0.69
Having to learn difficult topics in mathematics does not worry me.	15	0.67	0.75
No matter how much I study, mathematics is always difficult for me.	8	0.75	0.81
I have a lot of confidence when it comes to mathematics.	17	0.74	0.79
Mathematics Motivation			
Mathematics is a subject I like.	1	0.75	0.85
Spending a lot of time on a mathematical problem frustrates me.	16	0.57	0.58
I don't understand how some people can get so enthusiastic about doing mathematics.	18	0.48	0.61
I can become completely absorbed doing mathematics problems.	4	0.77	0.76
If something about mathematics puzzles me, I would rather be given the answer than have to work it out myself.	13	0.47	0.38
I like to stick at a mathematics problem until I get it out.	11	0.68	0.64
The challenge of understanding mathematics does not appeal to me.	22	0.45	0.61
If something about mathematics puzzles me, I find myself thinking about it afterwards.	21	0.52	0.48
Mathematics Engagement			
I prefer to work on my own than in a group.	19	–0.10	0.16
I find working through examples less effective than memorising given material.	9	0.27	0.48
I find it helpful to test understanding by attempting exercises and problems.	20	0.88	0.84
When learning new mathematical material I make notes to help me understand and remember.	14	0.21	0.33
I like to revise topics all at once rather than space out my study.	10	0.12	0.20
I do not usually take the time to check my own working to find and correct errors.	2	0.14	0.31

Table 1: Questionnaire used in our study. For each of the three factors, the items, their loading (*Load 1*: based on all data, *Load 2*: based only on paired data) and their position in the questionnaire (*Pos*) is given.

a person not taking part in the project respectivley, and $1 \leq j \leq 21$. After the above mentioned transformation, $X_{i,j}$ takes values in $\{0, 25, 50, 75, 100\}$. We are interested in how the students' attitude may have changed during the semester in both groups. Thus, for a general overview, we collect at first all scores in a factor and consider for each person i the

$$\text{individual increase in summarized factor } F = \underset{j \in J_F}{\text{mean}} \left(X_{i,j}^{(\text{post})} \right) - \underset{j \in J_F}{\text{mean}} \left(X_{i,j}^{(\text{pre})} \right),$$

where J_F is the set of all items that belong to factor F (i.e. either the factor *confidence* or *motivation*). A boxplot of the n_p and n_c values of this individual increase is shown in Figure 4, for both factors and both for the participants and for the control group. For a comparison of the increase in the single groups, the average pre value

$$\frac{1}{n} \sum_{i=1}^{n} \underset{j \in J_F}{\text{mean}} X_{i,j}^{(\text{pre})}$$

is also given for each group ($n = n_p$ or $n = n_c$).

For a more detailed analysis we also consider in a second step for each person i and some selected items j the

$$\text{individual increase in item } j = X_{i,j}^{(\text{post})} - X_{i,j}^{(\text{pre})}.$$

An overview is provided by Figure 5. Here, the mean and 95% confidence limits are given instead of the median and the quartiles in the boxplot, because the data is discrete on the set $\{0, 25, 50, 75, 100\}$ and both groups rate the items rather similar such that a boxplot becomes degenerate and does not carry much information (for the individual increase in the summarized factor above, this problem does not occur because of the averaging). For a comparison, also the avarage pre value

$$\frac{1}{n} \sum_{i=1}^{n} X_{i,j}^{(\text{pre})}$$

is given for each item j and each group ($n = n_p$ or $n = n_c$).

We have also analysed the increase in the factors and items formally. To this end, we have at first checked if each data set (pre/post and participants/control group for each factor) is normally distributed, using the Shapiro-Wilk test on a significance level of 5%. (All tests in this study have been done on this significance level, which will not be mentioned any more in the following.) The Shapiro-Wilk test yielded that the null hypothesis of an underlying normal distribution can not be rejected, see Table 3.

Second, we have performed Levene's test which ensured that for each factor, all four data sets (pre/post and participants/control group) exhibit the same variance. Finally, we have analysed the increase (the difference between the post and the pre survey) for each group and each factor with Student's t-test (we obtained the same results from a Mann-Whitney-Wilcoxon test which does not require normally distributed data and which we performed as well). We have done the same statistical procedures on the data for the individual increase in the single items.

5.3 Results

5.3.1 Results on summarized factors

The results of our evaluation concerning the factors *confidence* and *motivation* are displayed in Figure 4.

- *Mathematics confidence.* In the participants' group, there is no change observable. In the control group, one can see a slight increase. As the different lengths of the whiskers indicate, there are a few participants who exhibit a decrease in confidence and a few control group students who show an increase. Due to the symmetric distribution of the middle 50% of

	complete data		paired data sets	
	pre	post	pre	post
confidence	0.844	0.898	0.881	0.905
motivation	0.811	0.821	0.827	0.850
engagement	0.326	0.035	0.272	0.340

Table 2: Cronbach's α for the three factors *confidence*, *motivation* and *engagement* of our questionnaire, both for the evaluation at the start (pre) and at the end (post) of the semester

	Shapiro-Wilk test				Levene's test
	part. pre	part. post	c.g. pre	c.g. post	
confidence	0.345	0.557	0.853	0.236	0.609
motivation	0.115	0.483	0.357	0.357	0.119

Table 3: p-values of the Shapiro-Wilk test (left) for different data sets (pre/post and participants/control group for each factor) and p-values of Levene's test (right) for each factor

both data sets around 0, we can state that there is no change in mathematics confidence, neither in the control group nor among the participants.

Note that the participants start on a considerably higher level. This indicates that either *MathePraxis* appeals mostly to students who are self-confident about doing mathematics or that being accepted at the project gives the students a confident feeling.

- *Mathematics motivation.* While the (anyway moderate) motivation in the control group decreases during the semester, we can observe an increase among the participants (who are higher motivated from the beginning). The change is gradual, but while it seems to be only a slight development in the middle of the control group, which heavily scatters around 0, the location of the participants' group is higher at all, so we conclude that *MathePraxis* is capable of maintaining and increasing motivation.

In Table 4, the results from a formal analysis of the data via Student's t-test are given. Even though the test rejects the null hypothesis (that there is no increase in the groups) in neither case, the p-values support our conclusions – one must keep in mind that the test is based on a small sample and thus tends to keep the null hypothesis.

5.3.2 Results on single items

The results of our evaluation concerning some single items are displayed in Figure 5. In Table 5, the p-values of the respective Student's t-test are given.

- *Good results (I can get good results in mathematics).* During the semester, the self-assessment does not change in both groups.
- *Afraid (I am more afraid of mathematics than of any other subject).* While the control group does not change, there is a slight increase in the fear of mathematics among the participants. Moreover, note that the participants report a very low level of being afraid of mathematics. While the regular mathematics course obviously does not change the students' opinion about the difficulty, we conjecture that the participants of *MathePraxis* have got a realistic impression of the complexity of real problems and applications after the project.

	confidence	motivation
participiants	0.952	0.365
control group	0.538	0.971
part.-contr.	0.710	0.584

Table 4: p-values of Student's t-test for a change in the location between the pre and the post survey of the participants' data (*participants*), for a change between the pre and the post survey of the control group data (*control group*) and for the difference between the increase of the participants and the increase of the control group (*part.-contr.*), each for the two factors *confidence* and *motivation*.

- *Want answer (If something about mathematics puzzles me, I would rather be given the answer than have to work it out myself).* Both groups start approximately on the same level, but after the semester, there is a clear difference: The control group reports a stronger desire to get the solution to mathematical problems without working for it, which is also reflected in the very low p-value of the t-test. Our application-orientated approach obviously maintains eagerness for deeper understanding.
- *No appeal (The challenge of understanding mathematics does not appeal to me).* While the control group is rather indifferent concerning mathematical understanding, the middle half of the participating students reports an (in parts heavy) increase of interest which can be seen in the result of the t-test as well.
- *Think afterw. (If something about mathematics puzzles me, I find myself thinking about it afterwards).* This item shows that the *MathePraxis* participants are interested in mathematical thinking, and they keep this interest over the project. In contrast, the control group looses its (anyway moderate) interest. This decrease is statistically significant, as Table 5 shows.
- *Excerc. help (I find it helpful to test understanding by attempting exercises and problems).* The project makes the participants realise the use of practical examples: the rather high level is even increased. The control group, which also started at a high level, sees less use in excercises at the end of the semester. This decrease is statistically significant, too.
- *Make notes (When learning new mathematical material I make notes to help me understand and remember).* The learning habits of the control group does not seem to have changed, while the participants are a bit more aware that notes support understanding. Due to the small samples and the variance of the scores, this interpretation can not be confirmed by the conservatory t-test.
- *No check (I do not usually take the time to check my own working to find and correct errors).* A clear difference is visible: while the control group stays more or less neutral with a slight tendency to do more error checking, the project participants report a heavy change from in average much to moderate error checking. We conjecture that this is due to self-confidence and a secure feeling of understanding the techniques: after the project, the participants do not seem to be as afraid of errors as in the beginning.

6 Conclusion

In engineering sciences, there is a strong need for a more application-orientated education in the first semesters to counter the high drop-out rate among engineering students. Many students

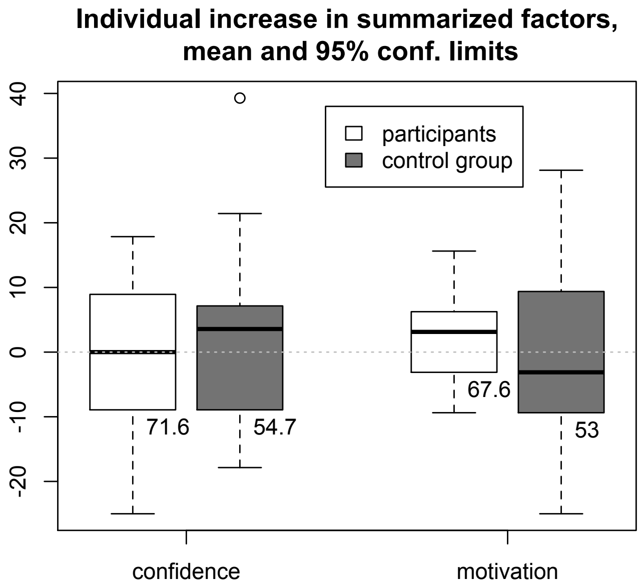

Figure 4: Boxplot of the individual increase (from pre to post) for the two factors *confidence* and *motivation*, each for the participants (white) and the control group (grey); below each bar, the mean start value (pre) is given.

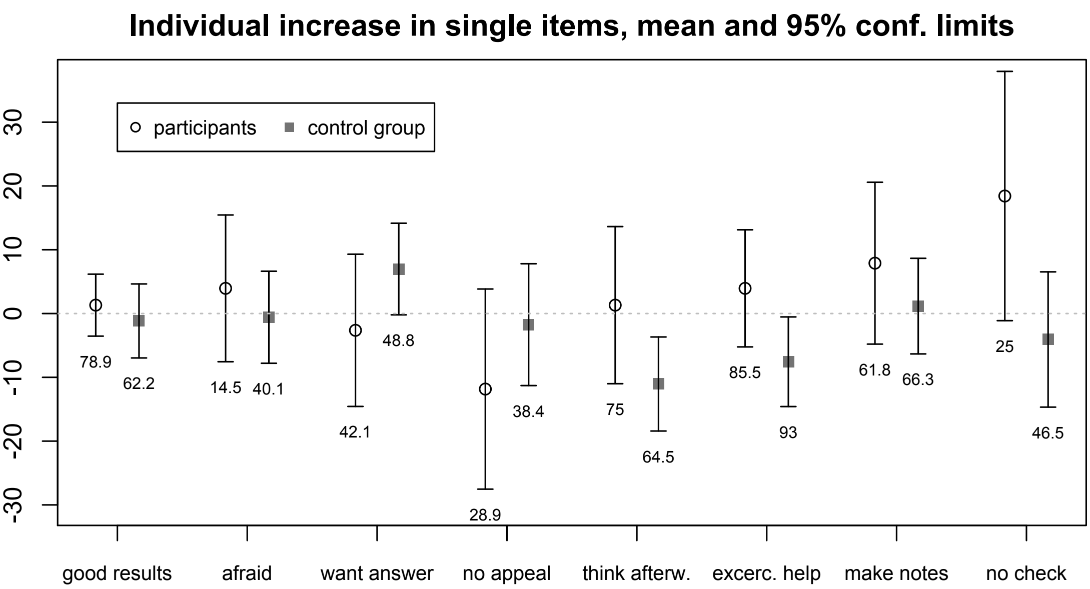

Figure 5: Individual increase (from pre to post) for different single items, each for the participants (white circle) and the conrtrol group (dark rectangle), mean and 95% confidence limits; below each bar, the mean start value (pre) is given.

	good results	afraid	want answer	no appeal
participiants	0.578	0.482	0.650	0.132
control group	0.688	0.872	0.057	0.714
part.-contr.	0.505	0.494	0.162	0.263

	think afterw.	excerc. help	make notes	no check
participiants	0.826	0.380	0.209	0.064
control group	0.004	0.036	0.756	0.443
part.-contr.	0.084	0.046	0.351	0.044

Table 5: p-values of Student's t-test for a change in the location between the pre and the post survey of the participants' data (*participants*), for a change between the pre and the post survey of the control group data (*control group*) and for the difference between the increase of the participants and the increase of the control group (*part.-contr.*), each for different single items.

complain about their abstract mathematics courses, but in our opinion, mathematics itself is not the problem, but the lack of appealing examples and application-orientated problems.

With our project *MathePraxis*, we tried to counter this. We showed that it is indeed possible to link basic mathematical topics with interesting and non-trivial examples from engineering, like studying differential equations with the help of a mass damper and an easy, but fascinating experiment by which the students can check their calculations and see the meaning of some abstract concepts like eigenvalues.

Our evaluation showed that this project is capable of maintaining interest and motivation, and it gives students a more realistic impression of the use of mathematics and the complexity of real problems. In our data, we cannot observe an effect on the mathematics confidence, and the increase of motivation is not high. We believe that this is due to the small sample size (which is intrinsical in this design as a test project), and maybe this is also caused by the design of the questionnaire which may be not completely appropriate.

It would be interesting to redo the project with a larger group of students (after the big interest in the actual run, we believe that most first-year students would like to take part in *MathePraxis* next year if it was offered) and to develop a questionnaire which is more aimed at evaluating if the students are motivated to continue their studies, if they are satisfied with the content of their lectures and if they wish to learn more about applications. We conjecture that this will confirm our approach – that giving students the opportunity to discover real applications of mathematical techniques with help of an experiment will support and motivate them such that their technical interest is satisfied.

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