MathePraxis – Connecting first year mathematics with engineering applications

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Abstract

First year engineering students often complain about their mathematics courses as the significance of the difficult and abstract calculus to their field of study remains unclear. We report on the project *MathePraxis*, a feasibility study which was designed as a means to give first year students some impression about the use of mathematics in real practice. We aim to increase motivation and retention rates among engineering students by connecting the contents of the first year mathematics lectures with practical applications. We developed three projects, two of which are described in this article: An inverted pendulum considered as a model for the automated control within a Segway and a study on the the optimal design of a ribbed cooler. In this article, we briefly present the mathematical content of the projects and report on their implementation.

Keywords: first year students; mathematics; motivation; practical orientation; segway; ribbed cooler; retention rates

1 Mathematics in Engineering education

1.1 Present situation

Mechanical engineering students in Germany typically begin their studies with at least two semesters getting mainly taught the basic knowledge in mathematics, mechanics and physics. The philosophy behind that structure is that only after students can master those basic skills they will be able to apply them successfully to problems from engineering. Unfortunately, a significant number of students struggle with this construction: After months of doing primarily other subjects they lose a lot of their former interest and sometimes even change their subject. Often they blame mathematics for their failure to complete their original goals. Observations however show that among those early university dropouts there are many students that would be able to finish their courses successfully if they could be motivated to learn the subjects.

A survey among more than 1000 German engineers [Pfenning and Renn, 2001] showed that retrospectively they considered their studies too abstract, not sufficiently team-oriented and far from practice. Even though there have been some changes in recent years, these issues are still a challenge for many engineering students.

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1.2 Real application and basic maths – incompatible?

Crucial questions therefore are if it is possible to connect first year mathematics with real engineering applications and if this is a way to maintain the students' motivation and interest.

Bringing in practical relevance into first year mathematics courses faces two essential roadblocks: First, real problems from an engineer's everyday work can in general not be solved with basic mathematics, and secondly there is hardly any time to embed examples in the mathematics lectures that take more than a few minutes, not to mention complex practice-oriented problems. Homework and excercises mostly concentrate on practising calculus schemes.

1.3 The *MathePraxis* approach

MathePraxis was created as a feasibility study to show that concrete engineering problems can be adapted from real requirements in practice to the students' level. It is a collaboration between the departments of Mathematics and Mechanical Engineering at Ruhr-University Bochum in cooperation with the center of higher education (IFB) and supported by Stifterverband für die Deutsche Wissenschaft and Heinz Nixdorf foundation. For details on the larger context and the accompanying project *MathePlus* see [Dehling *et al.* , 2010]. *MathePraxis* is an attempt to highlight the connections between first year mathematics and engineering applications. There have been different approaches to assist first year engineering students in mathematics by using more applications, see e.g. [Aroshas *et al.* , 2007], [Verner *et al.* , 2008] where short problems from various science and engineering domains were included in a multivariable calculus course in order to increase the internal motivation. We go one step further by using larger problems which cover several different topics at once. A number of groups suggest to include mathematical modelling in the first year curriculum, see e.g.[Huang, 2011] or focus combine mathematical questions with general problem-solving skills, see[Gleason *et al.* , 2010]. This however was not our intention in MathePraxis. Had we decided to focus on modelling of the engineering problems described below we would probably have spent all the available time to derive the model equations. For this reason we used topics which were suggested by engineers from the mechanical engineering department and adapted their mathematical details to the first year mathematics. Although we do not go as far as [Young *et al.* , 2011] where courses are taught by teams consisting of a scientist and an engineer the projects were at least established in close collaboration.

Based on the mathematics curriculum the projects should use only basic linear algebra and calculus of one or many variables. The three projects that were realized within *MathePraxis* so far deal with the control of a Segway, a sway control system for cranes and the effective CPU cooling by a ribbed cooler.

The rest of the paper is organized as follows. In section 2 we describe the general design of our course. Sections 3 and 4 give some impression about the mathematical contents for two of our projects while section 5 is devoted to the final presentation of the project results. In section 6 we comment on the students' performance as well as on their feedback.

2 Structure and benefits of the *MathePraxis* project

Students who had successfully passed the final exam of the first term could apply for *MathePraxis* by writing a short letter of motivation. The finally 28 participants then were given a practice-oriented problem which they should solve in small teams of 3–5 learners each during the 12 remaining weeks of the term. They were guided through the learning process by a combination of assignments (for examples, see sections 3.3 and 4.3) and some detailed text; This text, however, was not from a textbook, but the students had to discover the connection between the material, the mathematical formulas and real life on their own.

This course design is a variant of the *leittext method*, a method originally developed for industrial in-house training and described in [Rottluff, 1992],[Golle and Hellermann, 2000]. It allows for a self-regulated learning process by providing the learners with texts containing basic information, concrete questions or specific tasks. Based on these information, the groups have to plan their strategy, decide about which method to use and control their results afterwards.

At the beginning of all projects stood a short and interesting question: *Why doesn't a Segway fall over? How do the ribs of an optimal ribbed cooler look?* These questions adress the technical interest and purposefully omit mathematics. Although these problems in general require advanced knowledge and experience in mathematics, in *MathePraxis* they were refined and reduced so that students could solve them using only methods from their first year mathematics lectures, see Sections 3 and 4. Only where it could not be avoided, the student used numerical methods that had not been taught in the mathematics course. In the project, the participants did not learn new mathematical methods; they found out how standard and basic techniques can be used in practice. Note that this is in contrast to some other approaches like [Ooi, 2007] where the math courses are supplemented by examples in computational mathematics and the students have to learn to use a Computer Algebra System.

The teams could organise their working time on their own, with a weekly meeting with the project leaders. In this informal meetings of 45 minutes each, the students could ask questions or discuss about modelling and methods. We arranged these meetings primarily in order to ensure that the students which were not used to self dependent work stayed on the right way.

The weekly assignements were thought of both as a navigation and as a means to get to know the ideas behind a mathematical method. They consisted of specific tasks what to do next, to scrutinize the obtained results and to illustrate and to explain the approach and the used methods. During their first year at university, students often get the impression that mathematics occurs primarily in the form of clearly formulated exercises ("Solve the equation", "Calculate the integral over a given domain", . . .) with answers being either true or false. Working with the leittext assignments in addition involves problem solving abilities, deciding about solution strategies and evaluating results.

3 Balancing with differential equations – the Segway

The Segway is a stunning example for advanced and fancy engineering: It is a means of transport that can be found in everyday life, it is not too specialized or technical, it is fascinating and funny – and its construction would be impossible without mathematics. Thus a Segway is a shining example illustrating the need for higher mathematics in engineering education.

3.1 Setting the scene: The task

The real control systems in a Segway are much too complicated for first year students without any knowledge in automatic control, so the participating nine students (one group of five, one group of four persons) got the task to design a feedback control for an inverted pendulum, a pendulum which is mounted on a cart and deflected by an angle of 90° upwards (to the horizontal). Like a normal pendulum, the inverted pendulum is stable when hanging downwards, but it can be controlled by moving the cart horizontally in an appropriate way, so that the pendulum remains in its (unstable) upright position – like balancing an umbrella on the fingertip.

Scientific parlance
We had discussions with the students, if we can talk about *one* equation of motion although we have a system of *two* equations or that one variable $\vec{x}$ contains in fact several variables. *MathePraxis* contributes early to communication skills: Students learn to talk with each other about what they are doing and about subject-related conventions in notation and symbols.

3.2 Mathematical challenges

The single steps to develop a control for an inverted pendulum are:

Find the equation of motion of the uncontrolled inverted pendulum.

In principle, this is easy when using Lagrangian mechanics. Since at the beginning of the second semester, these techniques are not available yet, the equations of motion

$$\ddot{x} = -\frac{ml}{M+m}\dot{\alpha}^2 \sin\alpha + \frac{ml}{M+m}\ddot{\alpha}\cos\alpha + \frac{1}{M+m}F \tag{1}$$

$$\ddot{\alpha} = \frac{3}{4l}\ddot{x}\cos\alpha + \frac{3g}{4l}\sin\alpha \tag{2}$$

were given.

(1) and (2) look nasty and confusing, especially for first year students. In discussions we made clear that most of the variables (M, m and l) are known numbers (to simplify calculations, we later worked with concrete values). Some students had the (good) idea to plug one equation into the other in order to simplify the coupling, but this does not eliminate the nasty terms. The unpleasing conclusion was: Solving the equations of motion fails due to the non-linear behaviour of the pendulum.

Question the methods
A key aspect of *MathePraxis* is that the students learn to think when they apply calculus schemes, e.g. whether one can be content with a solution for the linearised differential equation: Is the linearisation – in whatever sense – "good enough"? Or when designing the control, why one chooses F to depend linearly on $\vec{x}$, and not in a more complicated functional relation.

Linearise.

In order to get rid of the non-linear terms, one has to linearise the equation of motion by a Taylor expansion. This provoked a first light bulb moment: The students knew Taylor expansions from their regular courses, but for most of them this was the first application of the method.

After linearisation, one obtains the equations

$$\ddot{x} = \frac{3mg}{4M+m}\alpha + \frac{4}{4M+m}F \tag{3}$$

$$\ddot{\alpha} = \frac{3g(M+m)}{l(4M+m)}\alpha + \frac{3}{l(4M+m)}F \tag{4}$$

Solve the system of linear ODEs.

We are interested in the behaviour of the point x and the angle α at time t. Combining these variables into

$$\vec{x} = \begin{pmatrix} x \\ \dot{x} \\ \alpha \\ \dot{\alpha} \end{pmatrix}.$$

one gets from (3) and (4) the linear ODE

$$\dot{\vec{x}} = \mathbf{A}\vec{x} + \vec{b}, \tag{5}$$

that is

$$\underbrace{\begin{pmatrix} \dot{x} \\ \ddot{x} \\ \dot{\alpha} \\ \ddot{\alpha} \end{pmatrix}}_{\dot{\vec{x}}} = \underbrace{\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{3mg}{4M+m} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{3g(M+m)}{l(4M+m)} & 0 \end{pmatrix}}_{\mathbf{A}} \underbrace{\begin{pmatrix} x \\ \dot{x} \\ \alpha \\ \dot{\alpha} \end{pmatrix}}_{\vec{x}} + \underbrace{\begin{pmatrix} 0 \\ \frac{4}{4M+m} \\ 0 \\ \frac{3}{l(4M+m)} \end{pmatrix} F}_{\vec{b}=\vec{b'}\cdot F}. \tag{6}$$

In this representation we clearly perceive that $\mathbf{A}$ describes the movement of the pendulum and the cart alone and that $\vec{b}$ contains the part we can influence via the force F. x and α are measurable output variables, F is the input. At first, we set $F = 0$ since we are interested in the uncontrolled movement of the system (the crucial point later will be to design F, depending on $\vec{x}$).

To solve this linear system of differential equations, we assumed to know the following exact values: the mass of the cart $M = \frac{1}{2}$ kg, the mass of the pendulum $m = 1\frac{3}{4}$ kg, the distance from the centre of mass, respectively the pivot point $l = \frac{1}{2}$ m and the acceleration of gravity $g = 10\,\mathrm{m}/\mathrm{s}^2$. Now $\mathbf{A}$ is quite simple.

The mathematics needed to solve this is covered by the course "mathematics for engineers" and most students almost immediately obtained the correct solution for $\dot{\vec{x}} = \mathbf{A}\vec{x}$:

Interpret formulae
In *MathePraxis* we emphasized descriptive understanding of equations, because this usually gets a raw deal in basic math education.
For example the strange terms e^{6t} in (7): The cart moves on and the pendulum rotates increasingly fast? This is hardly conceivable, so how to interpret this?
Of course, for the initial value $\vec{x}(0) = 0$, we receive what we expect: The pendulum remains motionless upright for all times. But if we allow even a tiny deflection, this solution is apparently wrong for large times.
This was a second light bulb moment: What the drawbacks of linearisation are.

$$\vec{x}(t) = \begin{pmatrix} c_1 - 7c_4e^{-6t} + 7c_3e^{6t} + c_2(1+t) \\ c_2 + 42c_4e^{-6t} + 42c_3e^{6t} \\ -18c_4e^{-6t} + 18c_3e^{6t} \\ 108c_4e^{-6t} + 108c_3e^{6t} \end{pmatrix}. \tag{7}$$

It took some time to guide the students to the understanding, that positive eigenvalues indicate instability of the system. This was a crucial point in the project: The students should acquire a link between an abstract equation and a movement in real life – an important skill which is hard to achieve by usual courses. Wherever possible, we highlighted these connections and practised with the students to interpret mathematical expressions, initial or boundary conditions, etc.

Understand how a state-feedback control works.

This requires of course an expertise which is available to first-year students. However, as the underlying idea is very intuitive, we just had to simplify notations and give the students a short introduction. We assume that the force F on the cart depends linearly on the state vector $\vec{x}$, i.e. $F = -\mathbf{K}\vec{x}$, where the control matrix $\mathbf{K}$ specifies, how exactly. In our special situation, a system with only one input variable, $\mathbf{K}$ is a row vector. Now (5) becomes

$$\dot{\vec{x}} = \mathbf{A}\vec{x} + \vec{b} = \mathbf{A}\vec{x} + \vec{b'}F = \mathbf{A}\vec{x} - \vec{b'}\mathbf{K}\vec{x} = (\underbrace{\mathbf{A} - \vec{b'}\mathbf{K}}_{\mathbf{A'}})\vec{x}, \tag{8}$$

The behaviour of the controlled system is obviously determined by a linear differential equation system $\dot{\vec{x}} = \mathbf{A'}\vec{x}$ with the system matrix $\mathbf{A'} = \mathbf{A} - \vec{b'}\mathbf{K}$. Now the art of a good control is to find a $\mathbf{K}$ such that the system is stable. In discussions we pointed out that in order to achieve this, the new system matrix $\mathbf{A'} = \mathbf{A} - \vec{b'}\mathbf{K}$ must not have positive eigenvalues. The objective then is to achieve this by choosing the entries of $\mathbf{K}$ (this is the so called *pole placement*).

Find an appropriate control.

Unfortunately there does not exist a simple and satisfying procedure the students could perform to obtain a pole placement. In `MATLAB`, the implemented command `K = place(A,B,p)` produces a feedback gain matrix $\mathbf{K}$ which gives $\mathbf{A}'$ any desired eigenvalues (collected in a vector p), but as first year students often are not familar with computer algebra systems, we decided that only for one black box effect it is not worthwhile to spend time on it. So we let the students try different $\mathbf{K}$ to get a feeling for how $\mathbf{K}$ influences the eigenvalues of $\mathbf{A}'$.

3.3 Excercises

Even though we wanted the students to collect and process information independently we gave them exercises to guide them along general lines and to help them to develop an understanding of concepts, perceptions and terms. These excercises were designed by the project leaders in close collaboration with university didacts. Some examples:

- Make a presentation for non-engineers: What is a Segway? How does it work? The presentation should take five to ten minutes.
- Write an excursus for a schoolbook (used in an advanced class in math): What is an equation of motion? What is a differential equation? Where do both occur in an engineer's workday life? Give some simple examples!

3.4 The action day

In order to keep motivation high over the weeks and to combine theory and real life, thinking and sensation, we made a day of action in the middle of the semester on which the students could have a ride on some rented Segways to check out: How it feels when one shifts one's weight on the platform and the Segway establishes a corresponding speed to keep the balance and how fast and how robust the control is.

After a short introduction on a street with little traffic, we made some round trips over the whole university campus. This was one highlight in the project: The students had a lot of fun testing the devices – standing still on the Segway, driving the first few meters and stopping are not trivial at the beginning. By the way, it was fascinating to observe how fast some of the students explored the limits of the control system.

4 Keep cool – the ribbed cooler

A cooler is a practically relevant example from the field of thermodynamics and was chosen to give students an idea of how heat transmission in general and especially in a ribbed body works. Cooler are objects from everyday life that can be found in computers, compressors of all types, car engines and many other objects.

4.1 Setting the scene: The task

The science of heat transmission tries to explain how the conducted heat flow depends on the impelling temperature gradient and how fast or intense the irreversible process of heat transmission proceeds. In our specific project, we focused on the process of heat conduction.

As *leittext* the participating students were given extracts from a frequently used textbook [Baehr and Stephan, 2003]. Their task was to construct theoretically a ribbed heat sink for a given setting (CPU-processor), after picking up the theory of stationary, geometrically one dimensional heat conduction.

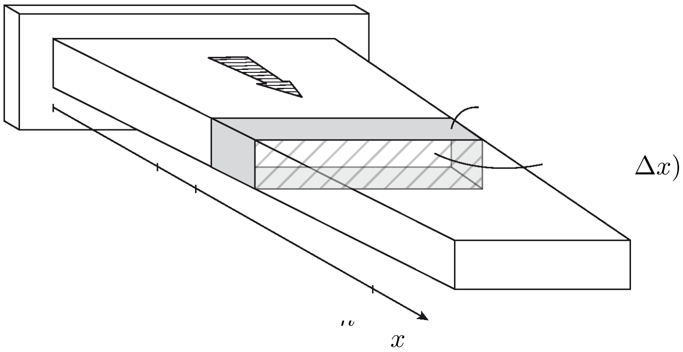

Figure 1: Energy balance for a volume element

4.2 Mathematical challenges

There are several steps to determine an "optimal" ribbed cooler:

Simplify the general heat conduction equation.

The starting point for the students was the general heat conduction equation

$$\rho c(\vartheta)\frac{\partial \vartheta}{\partial t} = \nabla \cdot (\lambda(\vartheta)\nabla\vartheta) + \dot{W}(\vartheta, \mathbf{x}, t),$$

where λ denotes heat conductivity, ϑ temperature and $\dot{W}$ heat source. Assuming that λ, the heat transfer coefficient α and $\dot{W}$ do not depend on temperature, the students were able to simplify this equation to

$$\nabla^2\vartheta + (\dot{W}/\lambda) = 0,$$

a differential equation, which they could easily solve for a constant source term $\dot{W}$.

Set up a heat balance for gills and needles.

Using figure 1 the students were able to deduce a differential equation

$$\frac{d}{dx}[A_q(x)\frac{d\Theta}{dx}] - \frac{\alpha}{\lambda}\frac{dA_R}{dx}\Theta = 0$$

which describes the behavior of the excess temperature $\Theta(x) = \vartheta(x) - \vartheta_a$, with ϑ_a being the ambient temperature. The excess temperature is an important factor in the equation describing the heat flow. Considering only straight gills with constant profile function, the equation above reduces to a homogeneous scond-order ordinary differential equation. The students could solve this and got an expression for the excess temperature.

Maximize the heat flow rate.

Using Fourier's law, the heat flow is given by

$$\dot{Q}_R = -\lambda_R b \delta_R \left(\frac{d\Theta}{dx}\right)_{x=0}.$$

With constant gill volume V_R and constant width b this can be rewritten as

$$\dot{Q}_R = \sqrt{2\alpha_R\lambda_R V_R b}\frac{\Theta_0}{\sqrt{h}}\tanh\left(\sqrt{\frac{2\alpha_R b}{\lambda_R V_R}}h^{3/2}\right),$$

where the only free variable is the height h. The students maximized $\dot{Q}_R(h)$ with respect to the height h via the equation

$$\tanh(mh) = 3mh[1 - \tanh^2(mh)] \text{ with } m = \sqrt{2\alpha_R/(\lambda_R\delta_R)}$$

which can be solved geometrically to $mh = \sqrt{2\alpha_R/(\lambda_R\delta_R)}h = 1.4192$.
This gives the optimal ratio between the height h and thickness δ_R of a gill.

Deduce the efficiency of gills.

The efficiency $\eta_R = \frac{\dot{Q}_R}{\dot{Q}_{R_0}}$ of a gill is defined as the ratio of the emitted heat flow $\dot{Q}_R$, compared to the heat flow $\dot{Q}_{R_0}$ emitted by a gill with uniform bottom temperature. For our case of straight gills with constant profile function

$$\eta_R = \frac{\tanh(mh)}{mh}.$$

The efficiency of a gill just depends on the proportion of height h and thickness δ_R.

Applying the theory to an example.

In the last step, the students were given a specific data like dimensions and material constants, and had to compute the optimal ratio of height and thickness for two different materials (aluminium and copper). At the end they computed the efficiency of their gill and looked up if such a ribbed cooler could be used for up-to-date CPU-processors.

4.3 Exercises

Similar to the Segway project, the students were guided by text extracts to the goal of computing an optimal heat sink. Some examples:

- Write an information box for a popular scientific journal:
  - How does the efficiency of a gill depend on parameters?
  - What does efficiency close to one or zero mean?
- Write a school book article for a mathematics/physics intensive course:
  - Which different kinds of ordinary differential equations do you know?
  - What do the notions $\nabla\cdot$ and ∇ in the general heat equation mean?
  - What is the definition of the geometrical one-dimensional, stationary heat conduction equation with heat source? Explain in your own words the statement of this heat conduction equation. What is the meaning of "stationary"?
  - Give an example from the everyday life of an engineer, in which the heat conduction equation plays an important role.

5 The final presentation

Each group prepared a presentation which was then shown to fellow students and professors from different departments. To this end the center of higher education at Ruhr-University Bochum offered a one-afternoon course in presentation techniques. It turned out that all groups had rather good prerequisites from their school education and none of the groups required additional assistance. The only problem they faced was the adequate depiction of mathematical formulas on Powerpoint slides. Two examples of these slides are shown in figure 2. Although the general level of the presentations was quite high, the technical level was very different. While one group focused

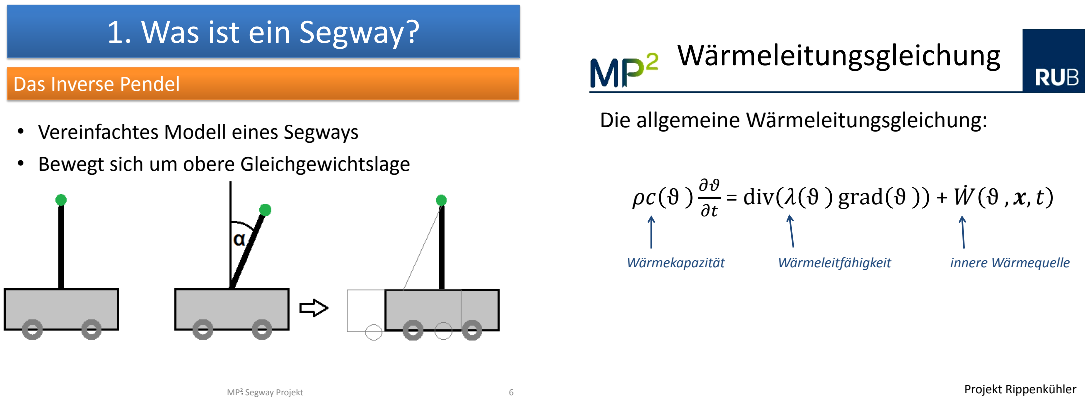

Figure 2: Sheets from the final presentation

on the history, construction and practical use of Segways another group only briefly mentioned this background information and quickly went over to the mathematical problems.

From our view the final presentation had two big advantages: First, the students were brought to think about the project as a whole and rethink their way of approaching the problem. Second, the final presentation can be thought of as some kind of summary of the project that they can later use as a reference. We therefore strongly agree with [Kågesten and Engelbrecht, 2007][Jeltsch and Nipp, 2002] who found that integrating such student presentations in a course supports the learning process.

6 Conclusions

There is no denying that in engineering education there is a need to be in step with actual practice. The project *MathePraxis* at Ruhr-Universität Bochum aims at giving students an impression of the use of basic mathematics in real engineering applications, beyond excercises and homework.

We found out that it is indeed possible to link subjects from the first year mathematics lecture to real and interesting applications. For example, the Segway is a bodacious possibility to connect differential equations, Taylor expansion and eigenvalues with up-to-date applications and with funny, impressive and tangible experiences. Even though we have shown that they can be refined to the level of knowledge of first-year students, it is in the nature of real problems that they require thoughts, time and efforts so that it remains to be investigated if these examples can be implemented in a regular mathematics lecture for first-year students. We also found out (by an evaluation, for details, see below) that the insight into applications in form of guided, but essentially single-handed teamwork leads to a more general and deeper understanding of basic techniques than can be achieved by regular courses. We believe that this increases students motivation to deal with mathematical subjects and that in this way, *MathePraxis* has a positive effect on retention rates among engineering students.

Our accompanying anonymous evaluation focused on two subjects: Several questions dealed with the students' beliefs about mathematics (see [Berkaliev and Kloosterman, 2009] or [Grigutsch and Raatz and Törner, 1998] for a similar survey with high school teachers), while in the other part of the questionnaire the students had to assess the significance of specific mathematical subjects for engineers.

The same questionnaire was given to the students at the beginning of the project and after the final presentation. A control group from fellow students who did not participate in *MathePraxis* was given the same questions at the same times. The five topics that were addressed showed up both in *MathePraxis* and in the general lecture.

We shortly report about the results since we believe that the students' attitude towards certain topics reflects their understanding and motivation.

It turned out that while in the control group the importance of the topic *Taylor expansion* decreased it did increase very significantly among project participants (see Table 1). Here the valuation of importance changed for the control group from 2.4 to 2.8 and from 2.7 to 2.0 for the project participants. For other topics like *derivatives* or *vector analysis* the difference was not as large but still the project members rated the importance of those topics higher than the control group. A detailed evaluation of the questionnaire will be given elsewhere.

Topic	CG before	CG after	PG before	PG after
Differential equations	2.4	2.4	2.2	1.8
Vector analysis	2.0	2.2	2.0	1.9
Eigenvalues	2.7	2.7	3.0	2.4
Derivatives	2.6	2.5	2.0	1.9
Taylor expansion	2.4	2.8	2.7	2.0

Table 1: *Mean values of the estimated relevance for different topics before and after MathePraxis. Note that smaller numbers indicate a higher importance of the topic. CG = control group; PG = project group*

We also made a short evaluation about the satisfaction with *MathePraxis* in the middle of the project; the students were asked to write down what they liked up to that point and how they would improve the project. The evaluation unambiguously showed, not surprisingly, that the participants liked the action day most; they recommended to add some more hands-on experience like designing a simple sway control or a LEGO Mindstorms realization of the inverted pendulum. The only aspects the students criticized were that the work load was not well-balanced and that the project "appears to be unstructured".

We believe that this originates in the fact that the students never experienced such independent work. In addition, the choice of the word "appears" indicates that the students actually realize that it is in the nature of real life problems that one has neither an overview nor a general idea what to do. Concerning the workload we recognize the difficulty of distributing the topics in a more balanced way: A short presentation over a Segway and a linearisation are just easier than solving a system of differential equations using generalised eigenvectors. In the planned rerun of the project we will try to account for that objection. We also plan to involve little practical realizations in order to add a creative aspect.

It is obvious that students have to put some time and effort into *MathePraxis*, but nonetheless, the mathematics involved in our project is covered by the regular lectures, and so *MathePraxis* was not exceptionally demanding. Since the work groups were small and we could monitor the students' progress in the weekly meetings and because we checked in interviews after the final presentation the students' understanding of the presented topics, we can be sure that even the weaker students could benefit from the project. We clearly see a need for reflecting methods by applying them in a more advanced context than in short homework excercises. This impression is supported by many intense and vivid discussions with the students during the project.

Especially during their first year at university, engineering students seem to have a strong interest in practical realisations, while they often neither see nor sense the mathematical abilities needed to realise them even if they are presented in the math courses. We believe that our project can close this gap and that it enables the students to get a first impression on how mathematics is used by engineers.

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