Dynamic factor demand in a rationing model

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In this paper, a dynamic decision model of the firm with a delayed adjustment of employment and investment is developed. Special attention is devoted to dynamic inefficiencies, i.e. underutilizations of the capital stock and labour hoarding. Market disequilibrium is introduced by allowing for a sluggish adjustment of wages and prices. The model of the firm is complemented by explicit aggregation, and the aggregate model is estimated for the FRG for the period 1960 to 1989. The empirical results reveal that dynamic adjustment constraints for employment and capital contributed to the persistence of unemployment in Germany in the 1980s.

I. INTRODUCTION

Despite some very interesting developments in the field of economic theory and econometrics, there is still a gap between theoretical models of economic behaviour and their empirical applications: most theoretical work is devoted to the analysis of equilibrium situations without saying very much about how this equilibrium is achieved; econometric practice, on the other hand, finds strong support for long lags in economic behaviour and pays special attention to the modelling of dynamic adjustment paths.

The contribution of this paper is the theoretical and empirical analysis of a dynamic adjustment model for employment and investment. In the model, uncertainty about demand shocks and a delayed adjustment of employment and investment are assumed. An explicit three-step adjustment process is developed which allows one to interpret the dynamic adjustment in terms of the theoretical model. There are two important outcomes of this approach: one is the consistent derivation of the slow adjustment of employment, investment, and capital—labour substitution with respect to changes in the economic environment. The other is a measure of the inefficiencies that are associated with the dynamic adjustment. A firm that cannot adjust the capital stock instantaneously will, on average, work with an underutilized capital stock or cannot satisfy demand; if the firm must choose employment before demand realization, labour hoarding can occur. Most emphasis is placed on the analysis of the medium-run employment adjustment. The model can explain the observed asymmetry of the employment adjustment during the business cycle: capacities and the labour supply place an upper bound on the employment adjustment in boom periods; additional asymmetries can arise from different constraints on the upward and downward adjustment.

Another issue is the consistent aggregation of microeconomic units. For instance, on the labour market some firms cannot realize their labour demand due to an insufficient labour supply, while others post ‘no-help-wanted’ signs due to high wages, a low demand for goods, or insufficient capacities. In consequence, at the aggregate level, unemployment and vacancies coexist. In the model here, employment is determined by the minimum of the labour supply and demand at the firm-level. Assuming a lognormal distribution function for firm level values, it can be shown that the aggregate transacted quantity can be approximated by a function solely in terms of aggregate supply, aggregate demand, and a mismatch parameter. The aggregate model is estimated for the Federal Republic of Germany. The estimation results reveal significant underutilizations of labour and capital during the business cycle. In addition, the dynamic adjustment of employment and capacities contributed significantly to the persistence of unemployment in Germany in the 1980s.

1See Lambert (1988) and Smolny (1993).
II. THE MODEL

Assumptions

The model developed here abstracts from an endogenous wage and price setting of firms. While an endogenous adjustment of wages and prices must be at the centre of any theory of inflation and income distribution, it is seen as less important for the quantity adjustment. In addition, the assumption of an immediate adjustment of wages and prices is equally restrictive.

It is further assumed that adjustment costs for employment and capacities depend solely on the delay between the decision to change a factor input and the completion of the adjustment: adjustment costs are negligible if firms take account of a factor-specific adjustment delay $\tau_i$, and prohibitive if firms try to adjust faster. This results in constant adjustment delays for labour and the capital stock. This kind of modelling of the dynamic adjustment takes account of the fact that changing decision variables necessarily takes time, and even a short delay between a decision and the realization of an exogenous variable can introduce considerable uncertainty. The analysis of the dynamic adjustment in terms of adjustment delays and uncertainty has the further advantage of reducing the dynamic decision problem of the firm to a sequence of static problems which can be solved stepwise.

(1) The short-run adjustment of output $Y$ can be analysed with predetermined employment and capacities. Output is given by the minimum of supply $YS$ and demand $YD$

$$Y = \min(YS, YD)$$

The production technology is approximated by a putty–clay production function with short-run limitationality and long-run substitution possibilities for labour $L$ and capital $K$

$$YS = \min(YC, YL) = \min(\pi_k K, \pi_l L)$$

$YC$ are capacities, $YL$ is the employment constraint of the production function, and $\pi_k, \pi_l$ are the productivities of labour and capital. The factor productivities are predetermined by the capital–labour ratio and the production function.

(2) The medium-run decision on employment with uncertain output and still predetermined capital stock and factor productivities. Employment is determined as the minimum of labour supply $LS$ and demand $LD$

$$L = \min(LD, LS)$$

(3) The long-run decision on the capital stock and capital–labour substitution with uncertain output and employment, taking into account the structure of the short- and medium-run decisions.

Optimal employment

Neglecting overtime work and inventory adjustment, the optimal output of the firm is given by Equations (1) and (2). The employment decision for time $t$ has to be made at time $t - \tau_i$, which means $\tau_i$ periods before demand realization; thus, it is made under uncertainty of demand. Labour supply for time $t$ is known in advance. The capital stock at time $t$ and the capital–labour ratio must be chosen before the employment decision, therefore capacities and labour productivity can be treated as predetermined. Optimal employment is derived from

$$\max_{-L} p_{-t} E(Y) - wL - cK + \lambda_{LS}(LS - LT)$$  \hspace{1cm} (4)$$

where $p$ is the output price, $w$ are wages, $c$ are the user costs of capital, $\lambda_{LS}$ is the shadow price of the labour supply constraint. $p_{-t} E(Y)$ is the expected value of output in $t$, with expectations formed at $t - \tau_i$. It is given by the expected minimum of supply and demand

$$p_{-t} E(Y) = \int_{YS}^{\infty} YD f_{YD} dYD + \int_{YS}^{\infty} f_{YD} dYD$$  \hspace{1cm} (5)$$

$f_{YD}$ is the probability distribution function (p.d.f.) of demand. Note that no explicit assumption about expectation formation is required. The first-order condition is

$$p \frac{\partial p_{-t} E(Y)}{\partial YS} \frac{\partial YS}{\partial YL} \frac{\partial YL}{\partial L} - w - \lambda_{LS} = 0$$  \hspace{1cm} (6)$$

The marginal increase of expected output from increasing supply is equal to the probability of the supply constrained regime\(^3\)

$$\int_{YS}^{\infty} f_{YD} dYD$$

The marginal increase of supply by increasing the employment constraint can take only the values 0 or 1: increasing employment will increase output supply only if there is capital to employ the additional workers. This implies that output supply is always given by the employment constraint of the production function. The third term is the predetermined productivity of labour. The first order condition can be rewritten as

$$p \text{prob}(YD > YS)(1 - \lambda_{YC})\pi_l - w - \lambda_{LS} = 0$$  \hspace{1cm} (7)$$

$\lambda_{YC}$ is the shadow price of the capacity constraint. Three cases can be distinguished.

\(^2\)Price adjustment in a rationing model is analysed in Smolny (1994).

\(^3\)Note that the value of both integrands in Equation (5) is equal.
Dynamic factor demand in a rationing model

(1) No constraints. $\lambda_{YC} = \lambda_{LS} = 0$. The firm is neither constrained by the available labour supply nor by the existing capital stock. Equation (7) reduces to

$$p \, \text{prob}(YD > YL^*) \pi_t = w$$

(8)

expected marginal returns marginal costs

The optimal probability of the supply constrained regime is equal to the full employment labour share $sl$. Optimal employment is determined as

$$L^* = F_{YD}^{-1}(1 - sl) / \pi_t \quad \text{with} \quad sl = \frac{w}{p \pi_t}$$

(9)

$F_{YD}^{-1}$ denotes the inverse of the cumulative distribution function (c.d.f.) of demand. Optimal employment is determined by the productivity of labour, the share of labour costs in nominal full employment output, and the parameters of the distribution function of demand. The partial derivative of employment with respect to the real wage is negative. The effect of a higher productivity of labour on optimal employment is ambiguous: there is an employment-increasing effect on profitability, but there is also a decreasing effect, since less labour is required to produce a given output. The effect of a higher expected demand on employment is positive; increased uncertainty, i.e. a higher variance of demand may increase or decrease employment, depending on the labour share and depending on the particular form of the distribution function of demand. For a lognormal distribution of demand, employment follows from

$$\ln L^* = -\ln \pi_t + \ln \left(\frac{1}{\gamma_2} \right) E(YD) - 0.5 \sigma_{\ln YD}^2 + \sigma_{\ln YD} F^{-1}(1 - sl)$$

(10)

$\sigma_{\ln YD}^2$ is the variance of demand, and $F^{-1}$ is the inverse of the cumulative standard normal distribution function.

A graphical interpretation is depicted in Fig. 1. The upper figure is the density function of demand, the lower figure shows marginal costs and expected marginal returns. Marginal costs are constant. The expected marginal returns of labour are decreasing for increasing employment due to the increasing probability of demand constraints on the goods market. For high values of employment, prob$(YD > YL^*)$ approaches zero, a unique optimum is therefore assured for $sl < 1$.

(2) Capacity constraint. $\lambda_{LS} = 0$; $\lambda_{YC} \neq 0$. The firm is rationed by the existing capital stock. This constraint influences optimal employment negatively as no more workers will be demanded than can be employed by the (predetermined) capital stock.

$$L_{YC} = YC / \pi_t = \pi_d K / \pi_l$$

(11)

Fig. 1. Optimal employment

Note that this implies that output supply is always determined by the employment constraint of the production function.

(3) Labour supply constraint. $\lambda_{LS} \neq 0$, $\lambda_{YC} = 0$. In the final case of insufficient labour supply, the firm has not enough applicants to fill all vacancies. Optimal employment is equal to the labour supply

$$L = LS$$

(12)

The three cases can be summarized by a minimum condition for optimal employment

$$L = \min(L^*, L_{YC}, LS)$$

(13)

In the model, the utilization of labour varies pro-cyclically with higher utilization in the presence of positive (unexpected) demand shocks and, hence, a pro-cyclically varying measured productivity of labour. This property is in accordance with observed stylized facts and stands in contrast to conventional models of dynamic factor input adjustment, which allow for an immediate adjustment of employment and short-run substitution of capital and labour, thus implying an anti-cyclical movement of the productivity of labour. Optimal labour hoarding decreases with less uncertainty of demand and in the presence of labour supply and capacity constraints. The model can also explain the observed asymmetry of employment adjustment and the larger variance of employment changes during recessions: capacities and the labour supply place an upper bound on the employment adjustment in boom periods; no similar bound prevents a reduction of employment during downturns.

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4See Davis and Haltiwanger (1992).

5Note that the employment determination is consistent with an endogenous price setting, if prices are adjusted with the same or a larger delay as employment. See Smolny (1994).
Optimal capacities

The corresponding decision problem for the film concerning investment is to maximize expected profits with respect to the capital stock \( K \) and the capital–labour ratio \( k \). The investment decision has to be made with a delay \( \tau_k \) before output realization and thus with a delay \( \tau_k - \tau_l \) before employment determination

\[
\max_{-K,k} V = p_{t-\tau_k} E(Y) - w_{t-\tau_k} E(L) - cK
\]

(14)

The expected value depends on expected output and expected employment, and hence on the density function of \( YD \) and \( LS \). In addition, \( L^* \) is uncertain at the time of the investment decision. It is influenced by the optimal capital–labour ratio, but it also contains a stochastic component in the demand expectations formed at \( t - \tau_l \). Differentiation with respect to the capital stock yields the probabilities of the capital constrained regime on the goods and on the labour market

\[
p \, \text{prob}[\min(YD, YL^*, Y_L) > YC] \pi_k
\]

(15)

expected returns

\[-w \, \text{prob}[\min(LS, L^*) > L_YC] / \pi k - c = 0
\]

expected additional wage costs capital costs

Three effects of an increased capital stock can be distinguished. The first is the marginal increase of capital costs \( c \); second, there is the expected increase of labour costs due to a marginal higher capital stock, a higher capital stock increases employment only in the capital-constrained regime on the labour market; finally, there is the marginal increase of expected returns due to higher capital. Employment and the productivity of labour determine the optimal supply of goods; thus, a higher capital stock affects output supply only if it affects employment. This is the first condition for positive returns on capital; second, higher returns are achieved only if the firm can sell the product, i.e. if demand exceeds supply. Optimal capacities are obtained from

\[
YC = F_{Y}^{-1} \left( \frac{1 - sk + asl}{1 - sl} \right) \quad \text{with} \quad sk = \frac{c}{\mu \pi k}
\]

(16)

where \( a \) is the marginal change of labour hoarding due to a higher capital stock. \( F_{Y}^{-1} \) is the inverse of the c.d.f. of \( Y^* \):\( t-\tau_k E(Y^*) \) is the expected minimum of those constraints which may prevent the firm from full utilization of capacities. It is defined as

\[
t-\tau_k E(Y^*) = \min(YD, Y_L, YL^*)
\]

Whenever the actual values of these constraints exceed \( YC \), the firm works with full utilization of capacities and could earn more profits with higher capacities. If one of these constraints is less than \( YC \), the firm has excess capacities and the marginal product of capital is zero. Therefore, the decision on the capital stock can be seen as the optimal choice of the probability of being constrained by capacities, and the value of this probability is determined by the ratio of marginal costs and marginal full employment cash flows, which depends on \( sl \) and \( sk \).

Equations (16) and (9) reveal the similarity of the behavioural equations determining employment and capacities. Unconstrained employment \( L^* \) is determined by the expected constraint on the goods market and the share of wage costs in value added: optimal capacities are determined by the expected constraints on the goods and labour market and the ‘profitability’ of capital. For a lognormal distribution of \( Y^* \), capacities can be calculated from

\[
\ln YC = t-\tau_k E(Y^*) - 0.5 \sigma_{ln Y^*}^2 + \sigma_{ln Y^*} F^{-1} \left( 1 - \frac{sk + asl}{1 - sl} \right)
\]

(17)

\( \sigma_{ln Y^*}^2 \) is the variance of \( Y^* \), and \( F^{-1} \) is again the inverse of the c.d.f. of the standard normal distribution (see equation (10)). For the empirical estimation, it can be calculated for all observed values of the argument.

The second component of the investment decision is the choice of the optimal capital–labour ratio \( k \). If the production technology is approximated by a CES-production function with exogenous technical progress and constant returns to scale, the following condition can be derived

\[
k = \left( \frac{z}{1 - z} \right)^{\sigma} \exp[\gamma_l - \gamma_k] t (1 - \sigma)]
\]

\[ \times \left( \frac{c}{w + \lambda_{LS} \frac{t-\tau_k E(DUL)}{\mu \pi k}} \right)^{\sigma}
\]

(18)

\( DUL \) and \( DUC \) are the degrees of utilization of labour and capital, and \( \gamma_l \) and \( \gamma_k \) are the rates of labour and capital saving technical progress, respectively, \( z \) is the parameter determining the relative production elasticities, and \( \sigma = 1/\sigma - 1 \) with \( \sigma \) being the elasticity of substitution. \( \lambda_{LS} \) can be interpreted as the shadow price of labour supply constraints. The slope of the transformation curve is equal to relative factor prices, corrected for the expected degrees of utilization of both factors, and corrected for the shadow price of the labour supply constraint. The optimal capital–labour ratio is always higher in the presence of labour supply constraints. The underutilization of labour and capital exhibits the same effect as higher costs. Therefore, the faster adjustment of labour as compared with capital biases the substitution decision in favour of employment.

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6The complete derivations are contained in Smolny (1993).

7Note that Equation (18) is a structural form between endogenous variables.
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Table 1. Regimes on the goods and labour market

<table>
<thead>
<tr>
<th>Labour market</th>
<th>Goods market</th>
<th>YD &gt; YS</th>
<th>YD &lt; YS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L = LS )</td>
<td>( Y = Y_{LS} )</td>
<td>( Y = YD )</td>
<td>repressed inflation</td>
</tr>
<tr>
<td>( LS &lt; \min(L_{YC}, L^* ) )</td>
<td>( Y = YC )</td>
<td>( Y = YD )</td>
<td>capital shortage</td>
</tr>
<tr>
<td>( L = L_{YC} )</td>
<td>( Y = YL^* )</td>
<td>( Y = YD )</td>
<td>Keynesian unemployment</td>
</tr>
<tr>
<td>( L^* &lt; \min(L_{YC}, LS ) )</td>
<td>( Y = YL^* )</td>
<td>( Y = YD )</td>
<td>mixed</td>
</tr>
</tbody>
</table>

Regimes on the goods and labour market

In Table 1, the different regime constellations of the firm on the goods and labour market are summarized. While in the standard model of the New Keynesian Macroeconomics only three regimes are possible, the non-simultaneity of the output, employment, and capacity decisions introduce the possibility of three further combinations of output and employment constraints. If the firm has to decide on employment before output realization, this re-introduces the possibility of the underconsumption regime. Underconsumption is characterized by a rationing of the firm on the goods and labour market. If the firm can decide simultaneously on output and employment, only one of the constraints, labour supply or the demand for goods, can be the binding constraint. The other modification of the regime possibilities is related to the non-simultaneity of the employment and investment decision and the assumption of a putty–clay technology. The firm decides on employment after choosing optimal capacities. This allows one to distinguish two possible sources of demand constraints on the labour market. First, optimal labour demand does not exceed the available number of working places \( L_{YC} \). Second, unconstrained labour demand \( L^* \) depends on goods demand expectations and profitability. The complete set of regime probabilities can be derived from the first-order conditions and the parameters of the trivariate distribution function of \( YD, LS \) and \( YL^* \). Note that these are the optimal probabilities the firm chooses by deciding on capacities and employment. A situation of equality of supply and demand in expected values, i.e. \( E(YD) = YS \), has no special significance in the model and does not define an 'equilibrium'. The optimal probabilities, which define some kind of equilibrium, are determined by relative factor prices, the parameters of the density function, and the parameters of the production function.

Adjustment constraints for employment

Employment cannot exceed the labour supply. The labour supply consists of those already employed in the firm and job applicants. Therefore, it is reasonable to allow for a dependence of the current labour supply on the past employment level. This argument can be easily introduced into the employment decision by assuming a constraint on the adjustment speed of employment. For labour supply, this can be formalized as

\[
LS_t \leq \min((1 + \delta_a)L_{t-1}, \ell_S) \tag{19}
\]

Equation (19) reflects a constraint on the maximum rate of applications, \( \delta_a \), as well as on the absolute level of labour supply. Labour supply increases if the firm increases employment, but only until it reaches an exogenous level constraint \( \ell_S \). It seems to be important to allow for both kinds of constraints: in the short run and during periods of high unemployment, the number of applications within a time period restricts employment growth; in the long run and during boom periods it is plausible that a low level of labour supply prevents higher employment. A similar dependence on past employment can be stated for the demand for labour. Investments in firm-specific human capital, implicit 'full employment contracts', and reputation losses give rise to costs of dismissing workers and tend to restrict the downward adjustment of employment to normal fluctuations, i.e. quits and retirement

\[
LD_t = \max[LD^*_t, (1 - \delta^d)L_{t-1}] \tag{20}
\]

\( LD^*_t \) is the target level of employment to which the firm wants to adjust. The maximum condition implies a limit on the downward adjustment, and \( \delta^d \) is the maximal rate of downward adjustment of employment. If the costs of dismissing are prohibitive, \( \delta^d \) is the rate of normal separations. Thus, there are three restrictions causing employment to differ from the target level of labour demand: the level of employment is restricted by the exogenous level constraint on labour supply; the decrease of employment does not exceed optimal separations; the number of job applicants within a time period can be binding.

This model is related to the flow approach of the labour market (see Blanchard and Diamond, 1989): Equations (19) and (20) imply that employment growth depends on the rates of excess supply and excess demand on the labour market. Another property of this kind of introduction of adjustment constraints is the simple way to allow for an asymmetric adjustment of employment during the business cycle. For \( \delta^d < \delta^u \), the downward adjustment is more impeded than the upward adjustment; \( \delta^u = \infty \) (\( \delta^d = 1 \)) implies an unconstrained upward (downward) adjustment.

\(^a\)Note that in the dynamic model here, the optimal target level of employment differs from those derived in the static context, i.e. employment is not only determined by current constraints on adjustment, but depends also on expected employment changes.

\(^b\)See Davis and Haltiwanger (1992) and Pfann and Palm (1993) for the relevance of an asymmetric adjustment of employment.
The aggregation of micro-markets

The macroeconomic structure of the model relies on the concept of micromarkets introduced by Kooman (1984) and Lambert (1988). The aggregate goods and labour markets are divided into micro-markets with homogeneous labour and output on each micro-market but limited mobility between them. A micro-market is defined by a single firm operating on it. At the aggregate level, at every moment in time, different firms face different constraints. One way to derive a relation between aggregate quantities is to state a density function for demand and supply on the micro-markets. For lognormally distributed micro-markets, a very simple, CES-type analytical expression for the transacted quantity can be derived. Aggregate output is determined from aggregate supply and demand, and a mismatch parameter

\[ Y = \left\{ YD^{-\rho_y} + YS^{-\rho_y} \right\}^{-1/\rho_y} \tag{21} \]

where \( \rho_y \) is a mismatch parameter with \( \partial Y/\partial \rho_y > 0 \) and \( \lim_{\rho_y \to \infty} Y = \min(YD, YS) \). \( \rho \) depends merely on the uncertainty of demand at the time of the employment decision.

Employment is determined by the minimum of supply and demand, while labour demand, in turn, is given by the minimum of the capacity constraint and demand-determined employment. The distribution of the minimum of two lognormally distributed variables can again be approximated by a lognormal distribution, and aggregate employment is determined from

\[ L = \left\{ LS^{-\rho_m} + [L^*^{-\rho_y} + L^*_Y^{-\rho_y}] \rho_m/(\rho_y) \right\}^{-1/\rho_m} \tag{22} \]

Given the assumption of a lognormal distribution for the variables at the micro level, the aggregate counterparts of the behavioural relations for demand determined employment and capacities can also be derived. For a lognormally distributed variable \( x \), \( E(\ln x) = \ln E(x) - 0.5 \Var(\ln x) \) holds, i.e. the equation containing the aggregate variables has the same structure as those for the individual firms. The only difference is a change in the normalizing constant, which is affected by the variance of these variables on the micro-markets.

The aggregation procedure can also be applied to capture the constrained adjustment of employment. Equation (20) contains a maximum condition but the expected maximum of two lognormally distributed variables can equally be approximated by a CES-function. The only modification is given by the change in the sign of the \( \rho \)-parameter. Hence, the whole model can be estimated solely with aggregate data.

III. ESTIMATION RESULTS

Capital–labour substitution

The model is estimated with quarterly data from 1960 to 1989 for the private sector of the Federal Republic of Germany. It consists of four behavioural equations which are estimated by a two-step procedure. The first step consists of the determination of the optimal productivities of labour and capital. Observed productivities deviate from optimal ones by the respective utilization of the factor in question. This implies a relation between actual productivities, factor costs, the shadow price of the labour supply constraint, and the utilization of the factors. By using indicators for the degrees of utilizations of labour and capital, the equations can be completely expressed in terms of observable variables. An indicator for the utilization of capital is given by the business survey series on capacity utilization \( q \) for industry, published by the ifo-institute. The indicator for the utilization of labour is based on the correlation of the utilizations of labour and capital. The most important sources of underutilizations are unexpected demand shocks, with employment adjusting faster to those shocks. This is captured by a dynamic specification of \( q \).

The significance of these indicators provides a first test of the assumptions applied for the derivation of the model: it accentuates the importance of a delayed capital formation, and the underutilization of labour stresses the role of a delayed adjustment of employment and the capital–labour ratio. In addition, it is an indicator of price rigidities. With perfectly flexible prices, firms can always lower prices in the case of negative demand shocks. The estimated degrees of utilization of labour \( DU_L \) and capital \( DU_C \) are given by (standard errors in parentheses)\(^{13}\)

\[ \ln DU_C_t = 0.509 \ln(q_t/q_{t}^{\max}) \tag{23} \]

\[ (0.04) \]

\[ \ln DU_L_t = 0.444 \ln(q_t/q_{t}^{\max}) - 0.408 \ln(q_{t-1}/q_{t-1}^{\max}) \tag{24} \]

\[ (0.06) \quad (0.10) \]

\( q^{\max} \) stands for the observed maximum of the variable.\(^{14}\) Very significant coefficients were found for the utilization indicators. The estimated coefficients imply an average utilization of capital of about 95% and an average utilization of labour of about 97%. The implied amount of labour

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\(^{10}\)The property of lognormally distributed micro-markets is derived in Smolny (1993).

\(^{11}\)See Smolny (1993). Note that the \( \delta \) parameters must not be equal for the firms.

\(^{12}\)The estimation procedure has been proposed by Sneessens and Drèze (1986), and is applied in most of the work contained in Drèze and Bean (1990).

\(^{13}\)The complete estimation results are contained in Smolny (1993).

\(^{14}\)The observed maximum of \( q \) was in 1970. \( q \) does not exhibit a secular trend.
hoarding can be seen from Fig. 2. $L$ is actual employment and $L_Y$ is the number of workers which are necessary to produce output. The degree of utilization of labour corresponds to an average amount of labour hoarding of about 600,000 workers, and in recession periods, labour hoarding exceeds 1,000,000 workers.

### Output and employment

The optimal productivities are used for the calculation of the output and employment series. Output supply is calculated from employment and the optimal productivity of labour. For the calculation of $YD$, it is assumed that demand can always be realized by switching to the foreign markets in case of supply constraints on the domestic market. The excess demand for domestic products is then given by those imports, which are caused by supply constraints on the domestic market plus those part of exports, which are not carried out due to supply constraints of domestic firms. Trade equations are estimated with the most important determinants included, and contain also an indicator for the excess demand on the domestic market. This yields an estimate for the excess demand, and aggregate demand is calculated according to

$$
\hat{Y}D = YT + \hat{M}' (\text{excess demand})
+ \hat{X}' (\text{excess demand})
$$

with $X', M' \geq 0$. The output equation is estimated by nonlinear least squares, and the results are contained in Table 2.

### Table 2. Output

<table>
<thead>
<tr>
<th>AR(1)</th>
<th>AR(2)</th>
<th>AR(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.474 (0.09)</td>
<td>0.411 (0.09)</td>
<td>-0.200 (0.09)</td>
</tr>
</tbody>
</table>

SEE: 0.0023 BP(8): 21.6

Sample 1960.1–1989.4. Standard errors in parentheses. The estimation is carried out in logs. The equations include seasonal dummies, but no constant.

AR(n): coefficient of autocorrelation of order $n$.

Mismatch is specified by a linear and a quadratic time trend. The results imply a rate of excess demand and supply at $YD = YL$ of about 2% at the beginning of the observation period which increases to about 3.5% in 1989. This implies an increase of the importance of adjustment barriers of employment.

The basic model of employment determination was concluded by the minimum condition for employment. The estimation of the productivity and trade equations allows one to calculate all series required for the estimation of the employment equation. These series are seen as an important outcome of the approach. In Fig. 3, they are depicted together with actual employment. The most striking characteristic of

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Note that the standard errors are biased because $\hat{Y}D, \hat{Y}L$ are estimated series.

The Box–Pierce statistic reveals significant autocorrelation of the residuals. One possible source is the too simple specification of mismatch.

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The data depicted in the figures are seasonally adjusted by constant seasonal factors.

The method was developed by Sneessens and Drèze (1986). The results are taken from Smolny (1993).

A hat stands for an estimated series.

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19 The Box–Pierce statistic reveals significant autocorrelation of the residuals. One possible source is the too simple specification of mismatch.
demand determined employment is its high variance over the business cycle: during recession periods it is far beyond actual employment; in boom periods it increases faster than employment. Referring solely to the figure, employment adjusts slowly with respect to demand during the upswing and during the downswing. On the other hand, the development of capacity employment is smoother than actual employment, and \( L_{YC} \) lags behind employment. This indicates the slower adjustment of capacities with respect to demand.

The figure draws a rather detailed picture of the economic situation in Germany. Until 1966, an equilibrium situation can be stated. Labour supply was slightly below capacity employment, goods demand about equalled capacities, and the unemployment rate stayed at about 1%. Employment and the utilization of labour and capital remained fairly stable. This picture changed sharply with the recession in 1966. Demand determined employment decreased and capacities adjusted downward. However, the recession was only short-term and demand increased again until 1970. Capacities adjusted only slowly, and in 1970 the shortages of capital and the labour supply were the main factors restraining a higher growth rate of the economy. The German economy boomed when the first oil price shock hit the world economy. High inflation rates at the beginning of the 1970s induced the Deutsche Bundesbank to switch to a restrictive policy. High interest rates reduced investment and consumption demand, and exports declined in consequence of the slowdown of world demand. In 1975, the unemployment figure exceeded one million, and the utilization of labour and capital remained fairly stable. The partial recovery since then was terminated with the second oil price shock. The again restrictive monetary policy caused investment and consumption decreases in real terms, and the fiscal authorities changed to a restrictive course. In 1983, the unemployment figure exceeded two million. Since then, the economy switched on a path of sustained growth and the figures indicate that a higher employment growth at the end of the 1980s was mainly impeded by the slow adjustment of capacities.

In Table 3, the estimation results for employment are reported. Two mechanisms of the dynamic adjustment of labour demand (and supply) with respect to equilibrium values are tested. The first implies a lower and an upper bound on the adjustment speed of employment. Alternatively, a more standard specification of the dynamic adjustment of employment is tested, which is based on non-linear adjustment costs of employment instead of adjustment constraints. The adjustment path implied by non-linear adjustment costs is approximated by a partial adjustment model for labour demand.

The results reveal a constant mismatch. Allowing for a trend in \( \rho \) did not yield significant results for the dynamic CES-functions, which stands in some contrast to former estimates obtained for the FRG. However, note that one source of mismatch, i.e. adjustment constraints for employment, is taken explicitly into account here. The variability of employment is higher in the 1970s and 1980s than in the 1960s, therefore adjustment constraints are more important in this period. Second, no significant effect from the share of wage costs in value added on \( L^* \) was found. Real wage costs enter the employment determination mainly in the long run via capital–labour substitution and via capital formation. The dynamics yielded a remarkably better explanation of actual employment as compared with a static equation. The estimated adjustment is rather slow. In the CES-specification, the estimated coefficient for the upward adjustment is about 0.006 which implies that the firms, on average, cannot increase their labour force by more than 0.6% per quarter. It can be seen from Fig. 3 that the maximal observed adjustment speed is roughly in this dimension. The downward adjustment is impeded slightly less. The maximal downward

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**Table 3. Employment**

| Static CES-function |  
|---------------------|-------------------|-------------------|-------------------|-------------------|
| \( L_t = \{ LS_t^\rho + L_{YC}^\rho + \hat{L}_{YC_j}^\rho \}^{-1/\rho} \) | SEE: 0.0143 | DW: 0.538 |
| CES-adjustment |  
| \( L_t = \{ LS_t^\rho + LD_t^\rho \}^{-1/\rho} \) |  
| \( LS_t = \{ L^\nu + [(1 + 0.0062) L_{t-1} - \nu]^\rho \}^{-1/\rho} \) | (0.001) |
| \( LD_t = \{ LD_t^\nu + [(1 - 0.0101) L_{t-1}^\nu - \nu]^\rho \}^{-1/\rho} \) | (0.003) |
| \( LD_t^* = \{ L_{YC}^\rho + \hat{L}_{YC_j}^\rho \}^{-1/\rho} \) | \( 1/\rho = 0.0080 \) | \( 1/\rho' = 0.0194 \) | (0.001) | (0.005) |

| Partial adjustment |  
| \( L_t = \{ LS_t^\rho + LD_t^\rho \}^{-1/\rho} \) |  
| \( LD_t = 0.204 LD_t^* + (1 - 0.204) L_{t-1}^* \) | (0.023) | (*) |
| \( LD_t^* = \{ L_{YC}^\rho + \hat{L}_{YC_j}^\rho \}^{-1/\rho} \) | \( 1/\rho = 0.0057 \) | (0.001) |

Data sample 1960.1–1989.4. The estimation sample is shortened to allow for lags. Standard errors in parentheses. The estimation is carried out in logs. All equations include seasonal dummies, but no constant.

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20 See for example Entorf et al. (1990).
adjustment is estimated with about 1% per quarter. However, both coefficients are not significantly different from each other. Table 3 reports also the estimation results obtained from the partial adjustment specification of labour demand. The results are encouraging. Only two coefficients are estimated and the standard error of the equation is below 0.5%. The estimated dynamic adjustment is similar to those obtained from the pure CES specification, the adjustment coefficient $\lambda$ is about 0.2.\(^1\)

**Investment**

‘Time-series properties of investment, output and the cost of capital do not appear to be consistent with well-established theories of investment. The best predictor of investment is found to be its own past history.’ This somewhat resigned facet is drawn from a recent comprehensive study of investment behaviour of the OECD.\(^2\) The quotation illustrates the difficulty of finding a stable empirical relation between actual investment expenditures and the determinants expected from theoretical models. It emphasizes also the importance of a careful modelling of the dynamic adjustment, which is implicit in the statement that current investment is best predicted by past investment.

Before turning to the estimation results, the relation of the model here and other models of investment behaviour is discussed. In most models, the optimal capital stock is affected by capital–labour substitution, and therefore some kind of relative-price variable. Second, it depends on an activity variable, which is interpreted as ‘demand’ in Keynesian models and as the optimal output level in neoclassical models. Third, profits are introduced into the decision of the optimal capital stock by allowing for market interest rates and internal interest rates, or which place a bound on the external borrowing. Finally, empirical investment models allow for a slow adjustment of the capital stock with respect to optimal values.

One difference to those models is introduced by the activity variable $Y^a$. It depends on demand, and therefore has a similarity to traditional Keynesian models, but it is also affected by labour supply constraints. Especially for the situation in Germany in the 1960s and at the beginning of the 1970s, one can argue that the availability of sufficient labour was an important determinant of the optimal capital stock. If the capital–labour ratio can be adjusted only very slowly, and the results of the estimation of the productivity equations confirm this assumption, the optimal capital stock is bounded by the labour supply. There is also a difference to neoclassical models. There, output summarizes the optimal choice of the firm and is therefore an endogenous variable. It should be replaced by the corresponding exogenous variables for the estimation. Output is also endogenous for the firm in the model here. It is determined by supply and demand on the goods market, while supply, in turn, depends on capacities. Therefore, it is replaced by goods demand and labour supply constrained output, which do not depend on the capital stock at the firm level. Finally, the profitability variable has a rather different interpretation in the model here. It has nothing to do with financial constraints, rather it has been assumed that the firm can finance investment at the current interest rate on the money market. Profitability affects the optimal capital stock of the firm via the optimal probability of being capacity constrained in its output and employment decision, i.e. it is the main determinant of the optimal degree of the utilization of capital.

The results of the estimation of the investment equations are contained in Table 4. $Y^a$ is calculated from a CES-function depending on demand and the labour supply constrained output level. For the exogenous variables $Y^a$ and $f_{sk}$, expectations have been calculated by a procedure relying on rational expectation formation: the variables were estimated on a lagged information set, and the fitted values of these equations are used as the expected values, with expectations formed at those lags. Probably, all of these expected values play a role for the investment decision: different investment projects are carried out with different delays. By this procedure, only the relative importance of these delays can be determined. The results reveal a relative minimum of the standard errors at a rather short lag and another minimum at a lag of about two years, which corresponds closely to the expected delays in capital formation. While some investment projects can be started without very many planning delays, others can be carried out only with considerable delays. The results indicate the loss of information incurred by aggregation. While perhaps from disaggregated data a more distinct result can be obtained, the analysis of aggregate investment data allows only an estimation of the most important lags. Therefore, aggregate investment was separated into structures and equipment, and the same procedure was applied. This is only a small disaggregation, but the results indicate another problem. The estimated delays are very similar for structures and equipment, which indicates the simultaneity of investment decisions.

Significant effects from the activity variable on investment were found, and an about equal weight of short-term and long-term expectations is revealed.\(^3\) The effect of profitability on the optimal capital stock is not very well determined, but the estimated coefficient in the equation explaining investments in equipment is roughly in the range expected from the theoretical model. The coefficient can be

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\(^1\)A possible source of autocorrelation is again the simple specification of mismatch. Note again that the standard errors are biased.


\(^3\)The development of capital productivity was approximated by a time trend.
rates adjust only very slowly with respect to the adjustment coefficient is only half of this value. Investment is optimal values per quarter, and for structures, the estimated result is the very slow adjustment of the capital stock with explaining investments in structures. The most striking re-
corresponding coefficient is not significant in the equation interpreted as the average expectation error of Sample 1964.1.

A measure of the short-run excess supply on the goods market is provided by the utilization of labour. The medium-run supply conditions are determined by labour supply and capacities. On the labour market, ‘Keynesian’ labour demand and capacity employment can be deter-
mained in addition to the labour supply. The employment market is provided by the utilization of labour. The

Word accuracy: 94.6
unemployment in Germany in the 1980s. At the beginning of the 1980s, the demand breakdown in the course of the second oil price shock reduced employment. After the recovery of demand in 1984, employment growth was mainly impeded by adjustment constraints for employment; at the end of the 1980s, the slow adjustment of capacities constrained employment.

A significant increase of structural unemployment in the usual static sense is not revealed by the estimates. The only kind of mismatch which has increased during the observation period was the adjustment constraints for employment, which are more important in periods of rapid changes of demand than in 'equilibrium' situations like the 1960s.

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